LOAD TESTS ON A WOUND ROTOR POLYPHASE INDUCTION

MOTOR WITH UNBALANCED VOLTAGES ON THE STATOR

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Definition of Symbols Used

I_1	Stator current in amperes.
I_2	Rotor current in amperes.
r_1	Stator effective resistance in ohms.
r_2	Rotor effective resistance in ohms referred to the stator.
T	Developed torque in pounds feet.
P	Developed power in watts.
Z_ϕ	Magnetizing impednace.
X_1	Stator reactance in ohms.
X_2	Rotor reactance in ohms referred to the stator.
V_1^+	Positive-sequence component of applied voltage.
V_1^-	Negative-sequence component of applied voltage.
I_1^+	Positive-sequence component of stator current.
I_1^-	Negative-sequence component of stator current.
I_2^+	Positive-sequence component of rotor current.
I_2^-	Negative-sequence component of rotor current.
$r_1^+ = r_1^-$	Resistance of stator to positive- and negative-sequence voltages.
r_2^+	Resistance of rotor to positive-sequence voltage.
r_2^-	Resistance of rotor to negative-sequence voltage.
S	Slip.
P_2^+	Positive-sequence developed power.
P_2^-	Negative-sequence developed power.

THE POLYPHASE INDUCTION MOTOR

I GENERAL THEORY

A thorough understanding of the performance of an induction motor with balanced voltages on the stator is indispensable for an understanding of its performance when the line voltages on the stator are unbalanced. The first part of this paper will, therefore, deal with the general theory of induction motors under balanced voltage conditions on the stator.

The induction motor is of great importance since it is used more frequently than any other type of motor. Approximately nine out of ten motors of between one and one hundred horse power are polyphase induction motors. It is the simplest of all motors, is exceedingly rugged, has a high efficiency and reasonably good power factor. The fact that it possesses these good qualities is largely responsible for the fact that electric power is almost universally distributed by polyphase circuits.

II CONSTRUCTION

In principle the stator or primary of an induction motor is the same as that for a synchronous motor or generator. In fact if we remove the rotating field from a synchronous motor and substitute an induction motor rotor the machine will operate perfectly.

In the synchronous motor or generator the field is excited by direct current passed in from the outside. In an induction motor, on the contrary, the current on the secondary or rotor is induced and the rotor has no connection with any outside source of current.

III ROTOR CONSTRUCTION

Rotors of induction motors are of two types, the squirrel cage and the wound rotor type. In the latter, the rotor has a winding identical in principle with that of the stator. It is practically always wound three phase and the ends of the winding are connected to three slip rings with brushes resting upon them. The object of this construction is to make it possible to increase the resistance of the rotor cirucit when it is wished to do so. In this way the torque at starting and at low speeds can be increased.

By far the more common type of rotor is the squirrel cage. The thin iron discs of the rotor are punched with a large number of equidistant slots around the periphery, and in each slot is placed a copper bar. Two end rings are welded to the ends of the bars so that they are all interconnected at both ends of the rotor. Usually the bars are placed in the slots without any insulation whatever, as this has little or no effect upon the performance of the machine. In small motors, the entire rotor core is frequently placed in a mould and all the bars and both end rings are cast in one piece. The material used is an aluminum alloy.

IV WHY THE MOTOR RUNS

When polyphase currents are passed through the stator a rotating magnetic flux is set up. If we pass an alternating current through an ordinary coil of wire we, of course, set up an alternating magnetic field which varies in intensity and direction but does not change position. If, however, we pass polyphase currents into a properly wound stator we set up a field which remains practically constant in strength but rotates at synchronous speed. The stator may have any even number of poles and the rotating flux set up will have the same number of poles. It will always rotate at such a speed that it passes from one north pole to the next north pole in one cycle. In other words, it rotates at synchronous speed.

The graphical representation is given in Figure 1.

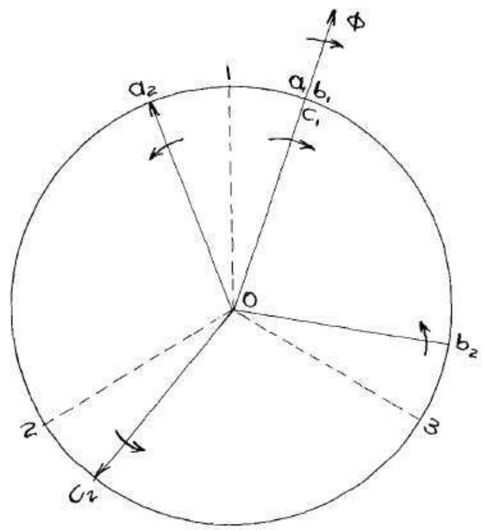


Figure 1

We have three fields along the lines O1, O2, and O3. If at a given instant the vector Oa_1 and Oa_2 are at the positions shown, giving the vectors Ob_1 and Ob_2 a start of 120 degrees and Oc_1 and Oc_2 a start of 240 degrees, they will be as represented. The vector sum of Oa_2, Ob_2, and Oc_2 is zero. Oa_1, Ob_1, and Oc_1 are in phase and their sum is three times the value of any one of them, giving the rotating flux as shown.

The flux density has approximately sinusoidal distribution in space and is represented in Figure 2 by the curve marked Bg.

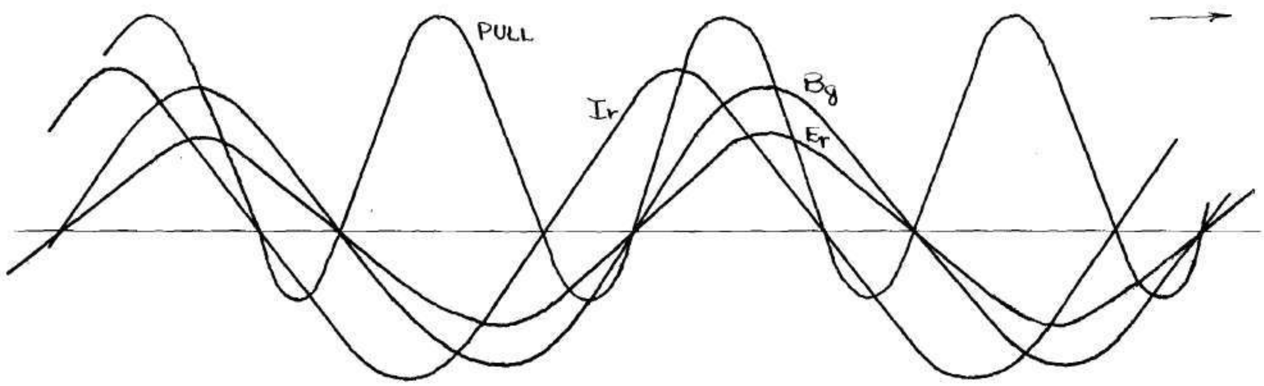


Figure 2

It should be carefully noted that the curves in this figure are space curves, not time curves. They represent the variation around the rotor of the flux density, e.m.f. and current at some particular time. As indicated by the arrow, the rotating field is supposed to be moving from left to right across the page. Assuming that the rotor is of the squirrel cage type, it is evident that voltages will be generated in the rotor bars, no matter what the speed of the rotor, unless it should happen that it is running at exactly synchronous speed. At any other speed, voltages are induced in the different bars and these voltages are represented by the distance from the base line to the curve marked Er. The ordinate of this curve at any point represents the voltage in a rotor bar at that particular point and time. This voltage is greatest when the flux density is greatest and, therefore, this curve is in phase with the curve of flux density B_g.

The voltages induced in the rotor bars cause currents to flow and since the rotor has inductive reactance, these currents will lag behind the voltage. They are represented by such a curve as I_r in Figure 2.

From the above exposition it can be seen that there are currents in the rotor bars lying in a magnetic field. Therefore, there is a force on each conductor tending to move it at right angles to the field and the current. The magnitude of the force is equal to

$$F = BIL \tag{1}$$

in which B is the flux density in lines per square centimeter, I the current in abamperes, L the length of the bar in centimeters, and F the force in dynes. The force on any bar is, therefore, proportional to the product of the flux density and the current. Multiplying these for successive positions the curve marked "Pull" is obtained. This curve represents, then, the pull on the different bars in the rotor.

Equation (1) can be readily changed to

$$F = \frac{8.87}{10^8} BIL \tag{2}$$

in which F is in pounds, B in lines per square inch, I in amperes, and L in inches. This expression is also true for instantaneous values of B and I.

The average value of this force is:

$$F = \frac{8.87}{10^8} \text{ BIL} \cos\theta \qquad (3)$$

in which B and I are the effective values of the flux density and the current.

V SLIP

In dealing with the induction motor frequent use is made of the term slip. By this is meant the percentage by which the speed of the machine differs from synchronous speed. It is equal to the synchronous speed minus the actual speed divided by the synchronous speed.

The voltage induced in a rotor bar is directly proportional to the slip, that is, to the difference in speed between the rotor and the rotating magnetic flux, or

$$E_R = SE_{RS}$$

where E_R is the e.m.f. in the rotor bar, S is the slip and E_{RS} the e.m.f. in the rotor bar when the rotor is standing still.

It will be readily apparent that the frequency in a rotor bar also varies with the slip. The frequency will be zero at synchronous speed and the same as the line frequency at standstill. Or

$$Fr = SF$$

in which Fr is the frequency in the rotor bar and F is the line frequency.

Since the reactance is also proportional to the frequency, the reactance of each rotor bar will also vary with the slip, or

$$X_R = SX_{RS}$$

where X_R is the reactance of the rotor bar and X_{RS} is the reactance of the rotor bar when the motor is at standstill.

An equation for the current in a rotor bar can now be readily written:

$$I_R = \frac{SE_{RS}}{\sqrt{R_2^2 + S^2X_{RS}^2}} \tag{4}$$

The angle of lag is

$$\cos \theta = \frac{R_2}{\sqrt{R_2^2 + S^2X_{RS}^2}} \tag{5}$$

It should be clearly noted that cos θ is the power factor in the rotor. The power factor of the entire motor is much lower.

VI RELATION BETWEEN TORQUE AND SLIP

If the assumption is made that the main flux of the motor, and consequently the flux density in the air gap Bg remains constant, one can readily derive equations showing how the torque in the motor varies with the slip and therefore with the speed. The assumption that the flux is nearly constant is based upon the fact that the back e.m.f. in an a-c device plus the XI and RI drops added vectorially must equal the line voltage. Actually in good motors the back e.m.f. and the flux remain nearly constant. However, since the torque varies as the square of the flux density, the following equations should be used only to give a general idea of what takes place. More exact calculations will be made a little later and the results compared.

It has been shown that:

$$\text{Torque} = K \times \text{flux density} \times \text{current} \times \text{const.}$$

The flux density in the equation can be replaced by something proportional to it, namely E_{RS}, of course changing the constant to correspond. Hence:

$$T = KE_{RS}\, I_R \cos\theta = KE_{RS}\, \frac{SE_{RS}}{\sqrt{R_2^2 + S^2X_{RS}^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + S^2X_{RS}^2}} = \frac{KSE_{RS}^2R_2}{R_2^2 + S^2X_{RS}^2}$$

Differentiate the torque with respect to the resistance:

$$\frac{dT}{dR} = \frac{K\, SE_{RS}^2\,(R_2^2 + S^2X_{RS}^2) - 2R_2 \cdot KSE_{RS}^2R_2}{(R_2^2 + S^2X_{RS}^2)^2} = 0$$

Simplifying, $S^2X_{RS}^2 = R_2^2$

$$S = \pm \frac{R_2}{X_{RS}} \qquad (6)$$

To get the maximum torque substitute in the equation for T and get

$$Tm = \pm \frac{KR_2E_{RS}^2R_2}{X_{RS} \cdot 2R_2^2} = \pm \frac{KE_{RS}^2}{2X_{RS}} \qquad (7)$$

VII TORQUE PROPORTIONAL TO ROTOR INPUT

The torque of an induction motor has been found to be:

$$T = KE_{RS}\, I_R \cos\theta \quad .$$

Since this is the product of the electromotive force induced in a rotor bar at rest, the current in the bar and the cosine of the angle between them, it represents the total input to the rotor when it is at rest, provided that K is the number of bars in the rotor. Hence one arrives at the important conclusion that the torque is equal to the rotor input in watts provided that it is expressed in synchronous watts.

The rotor I^2R_2 loss is readily obtained for the current from the equation:

$$\text{Rotor loss} = KI_R^2 R_2 = \frac{KS^2E_{RS}^2 R_2}{R_2^2 + S^2X_{RS}^2}$$

This expression is identical with that for the torque except that it contains the term S^2 while the torque expression contains only S. Accordingly:

$$\text{Rotor loss} = \text{Slip x Rotor input.}$$

This is a very important conclusion.

VIII CIRCLE DIAGRAM

If an induction motor is tested at various loads and the stator currents are plotted in their proper magnitude and phase angle, it will be found that the locus of the end of the current vector is approximately a circle.

$$\text{Rotor current } I_R = \frac{SE_{RS}}{\sqrt{R_2^2 + S^2X_{RS}^2}}$$

$$\sin\theta = \frac{SX_{RS}}{\sqrt{R_2^2 + S^2X_{RS}^2}}$$

Substituting the value of the radical in the above expression:

$$I_R = \frac{E_{RS}}{X_{RS}} \sin\theta$$

This is the equation of a circle whose diameter is $\frac{E_{RS}}{X_{RS}}$.

The circle diagram is relatively simple to construct from the no-load and blocked rotor tests and a complete analysis of the motor performance can be made from it. The results obtained check fairly closely with test results. In this paper the diagram will be used to check the performance of the motor under balanced voltage conditions.

IX MORE EXACT THEORY OF INDUCTION MOTORS

In Figure 3 is shown a combination of resistors and reactors which acts, as far as currents and voltages are concerned, just like an incution motor or a transformer with a noninductive load, assuming in both cases a one to one ratio between primary and secondary.

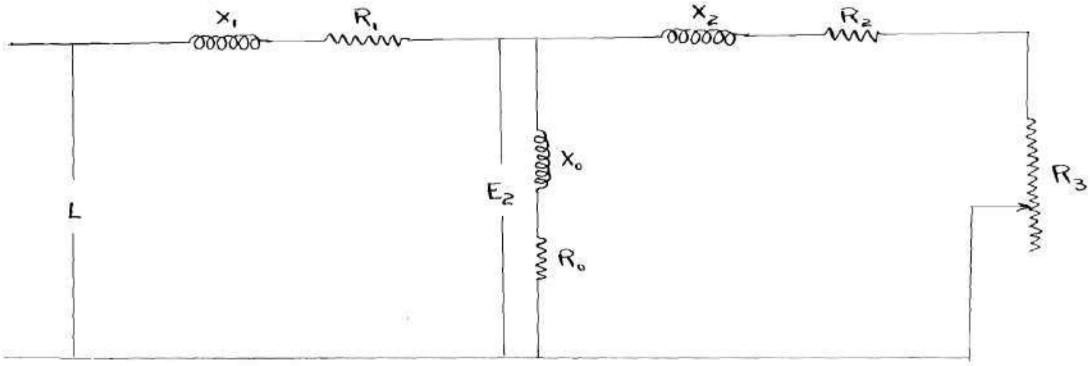


Figure 3

Rotor current $I_R = \frac{SE_{RS}}{\sqrt{R_2^2 + S^2X_{RS}^2}} = \frac{E_{RS}}{\sqrt{\frac{R_2}{S}^2 + X_{RS}^2}}$ as derived before.

In Figure 3

$$I_2 = \frac{E_2}{\sqrt{(R_2 + R_3)^2 + X_2^2}}$$

The above two equations are of the same form except that the equation for the motor has the expression $\frac{R_2}{S}$ and that for the network $R_2 + R_3$. Considering the usual range of the slip from 0 to unity, $\frac{R_2}{S}$ varies from infinity to R_2. Likewise if R_3 is varied from 0 to infinity, $R_2 + R_3$ varies from R_2 to infinity. Thus, varying the slip (i.e. changing the load) in the motor has the same effect as varying R_3 in the network, and any conclusions drawn from the network will be applicable to the motor.

The analytical treatment of Figure 3 is quite difficult. The equivalent circuit can be changed to that of Figure 4, considerably simplifying the treatment. Under certain conditions Figure 4 is more representative of motor performance than Figure 3.

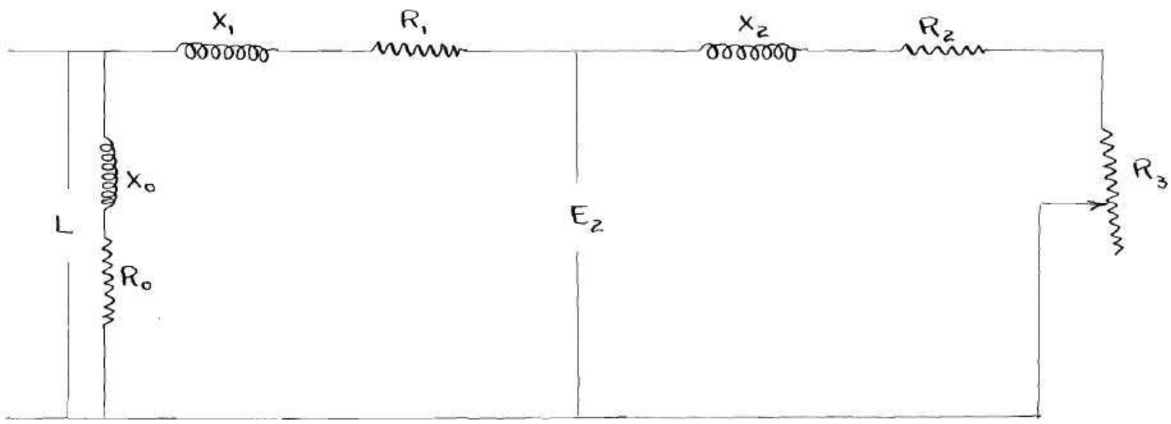


Figure 4

The torque in synchronous watts is equal to the rotor input. In Figure 4

$$\text{Rotor input} = T = I_2^2(R_2 + R_3) = \frac{L^2(R_2 + R_3)}{(R_1 + R_2 + R_3)^2 + (X_1 + X_2)^2}$$

As shown previously, one can make the substitution

$$R_2 + R_3 = \frac{R_2}{S}$$

Since R_3 is the variable, $R_2 + R_3$ can have any value from R_2 to infinity, and if the slip S is varied from 0 to 1, $\frac{R_2}{S}$ goes through the same values. Accordingly,

$$T = \frac{L^2 \frac{R_2}{S}}{\left(R_1 + \frac{R_2}{S}\right)^2 + \left(X_1 + X_2\right)^2} = \frac{SL^2 R_2}{\left(SR_1 + R_2\right)^2 + S^2\left(X_1 + X_2\right)^2} \qquad (8)$$

To find the maximum torque differentiate T with respect to R_2

$$\frac{dT}{dR_2} = L^2 \cdot \frac{S^3R_1^2 + 2S^2R_1R_2 + SR_2^2 + S^3(X_1 + X_2)^2 - SR_2(2SR_1 + 2R_2)}{\left[S^2R_1^2 + 2SR_1R_2 + R_2^2 + S^2(X_1 + X_2)^2\right]^2}$$

$$= 0$$

$$S^2R_1^2 + 2SR_1R_2 + R_2^2 + S^2(X_1 + X_2)^2 - 2SR_1R_2 - 2R_2^2 = 0$$

For maximum torque,

$$R_2^2 = S^2 \left[R_1^2 + (X_1 + X_2)^2\right]$$

$$S = \pm \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \qquad (9)$$

It has been shown before that for maximum torque:

$$S = \pm \frac{R_2}{X_2}$$

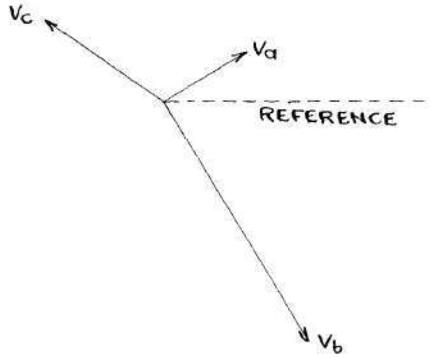
This equation has been derived from the simplified theory and is, therefore, less correct than equation 9, which shows that the primary reactance is equally as important as the secondary reactance in determining the maximum torque. In certain cases the two equations may give the same results, but in others, when R_1 and X_1 are large, the results may be widely different.

X THE INDUCTION MOTOR WITH UNBALANCED STATOR VOLTAGES

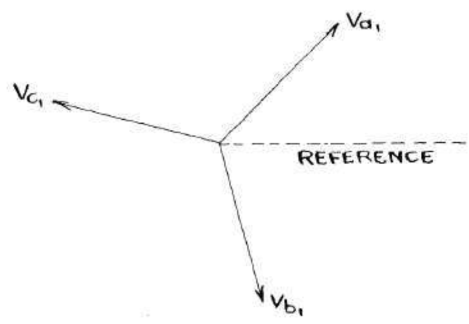
Polyphase induction motors are often required to operate from circuits in which the voltages on the different phases are unequal. This unbalanced voltage condition is usually found on polyphase circuits from which heavy single-phase loads are taken; it may result from unsymmetrical transformer connections or the use of unequal resistances in the motor leads to reduce the starting current. At first thought it might seem that a comparatively small voltage unbalance would have little effect on the operation of an induction motor, but when it is learned that 10 to 15% unbalance in voltage results in about 100% unbalance in current, it becomes very interesting to investigate what is happening to the motor characteristics when the stator voltages are unbalanced.

The study of the effect of unbalanced voltages has been greatly simplified by the demonstration that an unbalanced polyphase system can be resolved into two balanced systems of the same number of phases, the phase rotation of the one being counter to that of the other. This is accomplished by the method of symmetrical components which will be treated here briefly.

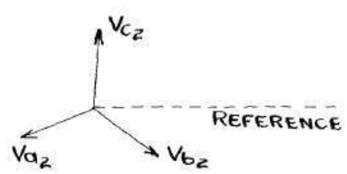
Any number of vectors up to and including three may be considered as an unbalanced system of three-phase vectors. The original vectors, V_a, V_b, and V_c are arbitrarily assigned the phase sequence abc in Figure 5a.



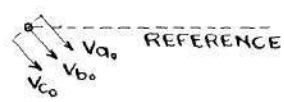
(a) ORIGINAL VECTORS



(b) POSITIVE-SEQUENCE VECTORS



(c) NEGATIVE-SEQUENCE VECTORS



(d) ZERO-SEQUENCE VECTORS

Figure 5

The balanced system of three-phase vectors that has the same phase sequence as the original system is called the positive-sequence system and will be denoted by the symbol V^+ as in Figure 5b. These vectors may be conveniently related to one another by means of the operator a, which is a unit vector 120° ahead of the reference axis. The operator a^2 is a unit vector 240° ahead of the reference axis. Thus, the positive sequence system may be written as:

$$V_a^+ = V_a^+$$

$$V_b^+ = a^2 V_a^+ = V_a^+ \;\angle 120^\circ$$

$$V_c^+ = a V_a^+ = V_a^+ \;\angle 240^\circ$$

The balanced system of three-phase vectors which is opposite in phase sequence to that of the original vectors is called the negative-sequence system. The vectors of this system will be denoted by the symbol V^- as in Figure 5c. They may be written as:

$$V_a^- = V_a^-$$

$$V_b^- = a V_a^- = V_a^- \;\angle 240^\circ$$

$$V_c^- = a^2 V_a^- = V_a^- \;\angle 120^\circ$$

The remaining system consists of three vectors, identical in magnitude and in time phase, as shown in Figure 5d. These vectors form what is known as the zero-sequence system and will be denoted by the symbol V_o. When the original unbalanced vectors add up to zero, the zero-sequence system is zero.

In Figure 5:

$$V_a = V_a^{+} + V_a^{-} + V_{a_o}$$

$$V_b = V_b^{+} + V_a^{-} + V_{a_o}$$

$$V_c = V_c^{+} + V_c^{-} + V_{c_o}$$

In terms of the operator a, the above relations may be written as:

$$V_a = V_a^{+} + V_a^{-} + V_{a_o}$$

$$V_b = a^2V_a^{+} + aV_a^{-} + V_{a_o}$$

$$V_c = aV_a^{+} + a^2V_a^{-} + V_{a_o}$$

Multiply V_b by a:

$$aV_b = a^3V_a^{+} + a^2V_a^{-} + aV_{a_o}$$

or, since $a^3 = 1$

$$aV_b = V_a^{+} + a^2V_a^{-} + aV_{a_o}$$

Multiply V_c by a^2

$$a^2V_c = a^3V_a^{+} + a^4V_a^{-} + a^2V_{a_o}$$

or, since $a^4 = a$

$$a^2V_c = V_a^{+} + aV_a^{-} + a^2V_{a_o}$$

Adding the equations for V_a, aV_b, and a^2V_c yields,

$$V_a + aV_b + a^2V_c = 3V_a^+ + (1 + a + a^2)\,(V_a^- + V_{ao})$$

Whence:

$$V_a^+ = 1/3(V_a + aV_b + a^2V_c) = 1/3(V_a + V_b \angle 120^\circ + V_c \angle 240^\circ) \qquad (10)$$

In a similar way:

$$V_a^- = 1/3(V_a + a^2V_b + aV_c) = 1/3(V_a + V_b \angle 240^\circ + V_c \angle 120^\circ) \qquad (11)$$

and,

$$V_{ao} = 1/3(V_a + V_b + V_c).$$

XI DEGREE OF UNBALANCE

When the voltages and currents in a polyphase circuit are unbalanced, it is sometimes convenient to express numerically, the "degree of unbalance" which may be said to exist in the circuit. At any point in a circuit the degree of voltage unbalance is given by the degree of negative-sequence unbalance and the degree of zero-sequence unbalance. Since the polyphase voltage system we are concerned with here has no zero-sequence components, we can define the unbalance factor as:

$$\text{Degree of Voltage unbalance} = \frac{V^-}{V^+} \qquad (12)$$

$$\text{Degree of Current unbalance} = \frac{I^-}{I^+} \qquad (13)$$

We are now ready to discuss the performance of an induction motor when the voltages impressed on the stator are unbalanced. When balanced sinusoidal voltages are impressed on the stator of an induction motor which has the usual symmetrical windings, a magnetic field is set up which is constant in magnitude and rotates at a constant speed fixed by the number of poles on the motor and the frequency of the applied voltages. This field is sinusoidal in its space distribution and can be represented by a rotating vector, which is equal in length to the maximum value of the flux density in the air gap and rotates at synchronous speed. The end of the vector, which represents the field, traces out a circle and the field is said to be circular. This was explained in the first part of this paper. The direction of rotation of the vector is fixed by the phase sequence of the applied voltages.

If the applied voltages are unbalanced, they can be resolved into positive-sequence and negative-sequence components by the method explained above. There can be no zero-sequence components, since the vector sum of the line voltages must be zero. Under these conditions, the rotating field in the air gap is no longer of fixed magnitude and it does not rotate at constant speed. It is the resultant, at each instant, of two circular fields produced by the positive-sequence and the negative sequence components of the impressed voltages. Each of these component fields rotates at synchronous speed with respect to the stator, but they rotate in opposite directions, due to the fact that the phase sequence of the negative-sequence components is opposite to that of the positive-sequence components. Except when the two fields are equal in magnitude, which occurs only for single-phase operation, the resultant of the oppositely rotating vectors, which represent the fields, traces out an ellipse and the field is said to be elliptical.

When the unbalanced voltages impressed on the stator have been resolved into the two balanced positive and negative voltage components, the performance of the motor can be calculated from the formulae developed in the first part of this paper for, first, the positively rotating voltage and, second, the negatively rotating voltage. These two performances can then be combined in their proper relations to obtain the resultant performance.

The constants which determine the operating characteristics of the induction motor are the stator resistance and leakage reactance and the rotor resistance and leakage reactance referred to the stator. In using the formulae developed before for the positive-sequence component of voltages, the slip S should be used, which is the actual slip of the motor under the given load conditions. However, for the negative-sequence component the slip 2 - S must be used. This is because the two fields are rotating in opposite directions.

The resistances of the stator to the positive-and to the negative-sequence systems will be denoted by $\overset{+}{r}_1$ and $\overset{-}{r}_1$ respectively, while those of the rotor will be denoted by $\overset{+}{r}_2$ and $\overset{-}{r}_2$. The resistances $\overset{+}{r}_1$ and $\overset{-}{r}_1$ are equal to each other and to the stator effective resistance, $\overset{+}{r}_2$ is the rotor effective resistance per phase at a frequency f_1S and is practically equal to the d.c. resistance. The resistance $\overset{-}{r}_2$ is the rotor effective resistance per phase at a frequency of $(2 - S)f_1$, which is about double the frequency impressed on the stator.

With unbalanced currents, the copper losses in the three stator phases will be unequal. One phase will be heated considerably more than when the motor delivers the same output under balanced voltage conditions. It is also found that the average of the unbalanced currents is always greater than the current per phase under balanced voltage conditions, which gives a higher copper loss in the stator, proportional to $I_1^{2+} + I_1^{2-}$. The total copper loss in the motor is

$$3\left[(I_1^+)^2 r_1^+ + (I_2^+)^2 r_2^+\right] + 3\left[(I_1^-)^2 r_1^- + (I_2^-)^2 r_2^-\right] \qquad (14)$$

In applying the torque equations, the calculated torque for the positive-sequence components of voltage must be considered positive, since it causes the rotor to run in the direction of rotation of its field and against the direction of rotation of the field of the negatively-rotating voltage. The calculated torque due to the latter system, therefore, must be considered negative.

Since the negative torque produced by the negatively rotating voltage substracts from the torque produced by the positive voltage, it represents so much lost torque and output, resulting in a lower efficiency. In order to compensate for this reduction in torque, the rotor must slip a greater amount to produce a given torque, resulting in a higher slip and consequently a higher I^2R loss in the rotor. There is also a tendency to increase the iron loss slightly due to the distortion of the field. These three increased losses combine to increase the input for a given output and reduce the efficiency.

It would be advisable at this point to review the formulae for the motor performance developed for balanced voltage conditions and show how they can be applied when the line voltages on the stator are unbalanced.

XII REVIEW OF FORMULAE

As explained above, when the voltages applied to the stator are unbalanced, two magnetic fields, of constant magnitude, are produced in the air gap and rotate in opposite directions. A vector diagram or an equivalent circuit can be drawn representing the conditions due to each of the two rotating fields. There will be a positive-sequence equivalent circuit and a negative-sequence equivalent circuit. The component parts of current, power, and torque can be calculated from the positive- and negative-sequence diagrams taken separately.

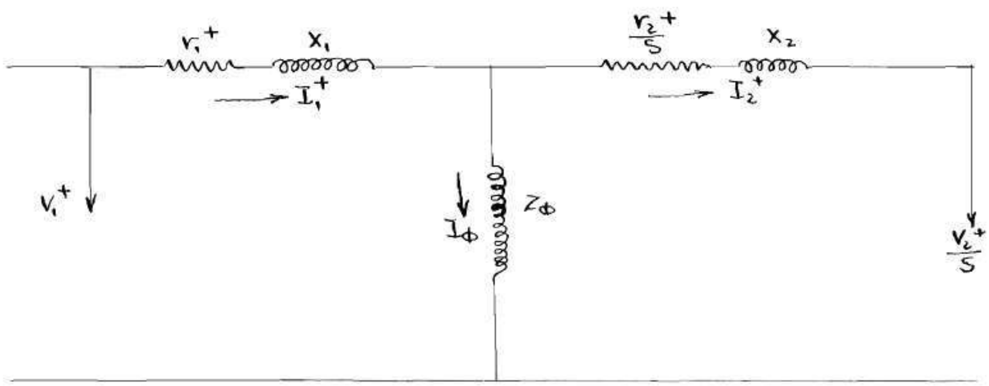


Figure 6

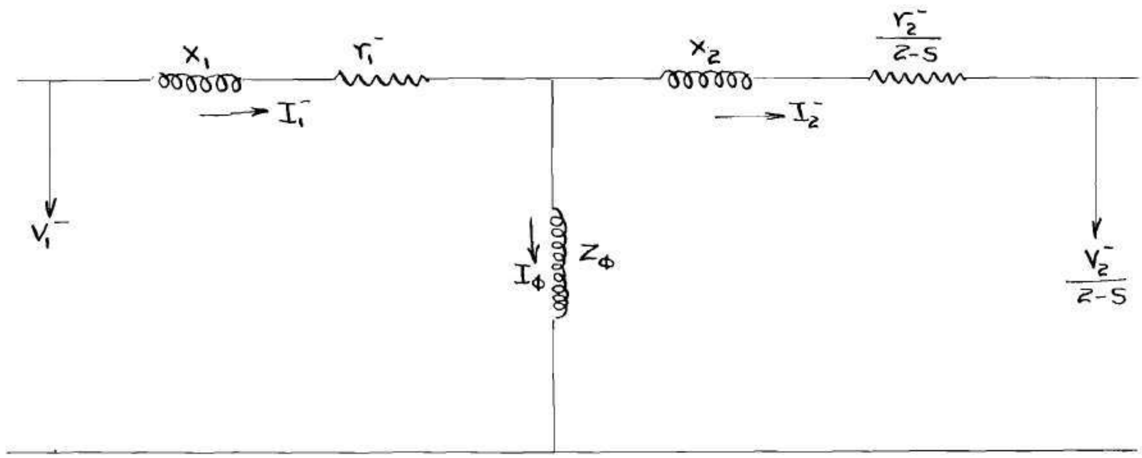


Figure 7

Assuming that the positive- and negative-sequence components of the unbalanced voltages applied to the motor are $\overset{+}{V}_1$ and $\overset{-}{V}_1$, respectively, the positive-sequence impedance of the motor is:

$$Z_1 = r_1 + jX_1 + \frac{(r_\phi + jX_\phi)(\frac{\overset{+}{r}_2}{S} + jX_2)}{r_\phi + \frac{\overset{+}{r}_2}{S} + j(X_\phi + X_2)} \tag{15}$$

Hence, the positive-sequence component of the stator current is

$$\overset{+}{I}_1 = \frac{\overset{+}{V}_1}{Z_1} \tag{16}$$

and the positive-sequence component of the rotor current is

$$\overset{+}{I}_2 = \frac{r_\phi + jX_\phi}{r_\phi + \frac{\overset{+}{r}_2}{S} + j(X_\phi + X_2)} \cdot \overset{+}{I}_1 \tag{17}$$

Similarly, the negative-sequence impedance of the motor is

$$Z_2 = r_1 + jX_1 + \frac{(r_\Phi + jX_\Phi)\left(\frac{r_2^-}{2 - S} + jX_2\right)}{r_\Phi + \frac{r_2^-}{2 - S} + j(X_\Phi + X_2)} \tag{18}$$

Hence, the negative-sequence component of the stator current is

$$I_1^- = \frac{V_1^-}{Z_2} \tag{19}$$

and the negative-sequence component of the rotor current is

$$I_2^- = \frac{r_\Phi + jX_\Phi}{r_\Phi + \frac{r_2^-}{2 - S} + j(X_\Phi + X_2)} \cdot I_1^- \tag{20}$$

From the above equations the following can be deduced immediately:

$$P_2^+ = (I_2^+)^2 \, r_2^+ \, \frac{1 - S}{S} \tag{21}$$

where P_2^+ is the positive-sequence component of the internal developed power per phase.

$$P_2^- = -(I_2^-)^2 \, r_2^- \, \frac{1 - S}{2 - S} \tag{22}$$

where P_2^- is the negative-sequence component of the internal developed power per phase.

$$T_2^+ = \frac{P}{4\pi f_1} (I_2^+)^2 \, \frac{r_2^+}{S} \tag{23}$$

$$T_2^- = -\frac{P}{4\pi f_1} (I_2^-)^2 \frac{r_2^-}{2 - S} \tag{24}$$

where T_2^+ and T_2^- are the positive- and negative-sequence components, respectively, of the internal developed torque per phase.

The total internal power and torque developed inside the motor can be written as

$$P_o = 3(P_2^+ + P_2^-) \tag{25}$$

$$T_o = 3(T_2^+ + T_2^-) \tag{26}$$

The total copper loss in the motor can be written as

$$\text{Cu loss} = 3\left[(I_1^+)^2 r_1 + (I_1^-)^2 r_1 + (I_2^+)^2 r_2^+ + (I_2^-)^2 r_2^-\right] \tag{27}$$

XIII ACTUAL TESTS MADE ON A POLYPHASE INDUCTION MOTOR

The actual tests on the induction motor were made for two purposes. In the first place it was desired to investigate the characteristics of the motor from actual test data when the voltages impressed on its stator are unbalanced. In the second place it was desired to find out how closely the actual test data would check with calculations made from the equivalent circuit of the motor by the method of symmetrical components.

The No Load Test

The motor was connected to a three-phase supply of rated voltage and then the voltage was varied over a wide range. At each point, the voltage, current, and power input were read, giving the curves shown on the curve sheets.

The no load power input represents core losses, friction and windage, and a small stator copper loss. By extending the curve of voltage versus power input to the zero voltage axis the friction and windage loss was predicted. The core loss is obviously the input at no load and rated voltage minus stator copper loss and the friction and windage loss.

The Rotor Blocked Test

The rotor was held firmly so that it could not turn, and a reduced voltage was applied to the stator. Voltage, current, and power were measured. The ampere reading is the short circuit current, and, because full voltage on the stator would cause excessive heating and mechanical stress, the applied voltage was less than the name plate rating. The relationship between applied voltage and current under these conditions are approximately linear, and by taking a series of meter readings with varying voltage a curve was plotted from which the short circuit current at rated voltage was determined. Watts input were also plotted against voltage but the curve could not be extended to give the watts input at rated voltage. Instead the in-phase components of the short circuit current were calculated at various points and plotted against voltage. From this curve the power factor at rated voltage was determined.

Load Tests

A load test was first run on the motor with balanced voltages applied to the stator. To vary the load a prony brake equipped with a dash pot was used. Voltage in each phase, current in each phase, watts input, torque output, and speed for various loads were read.

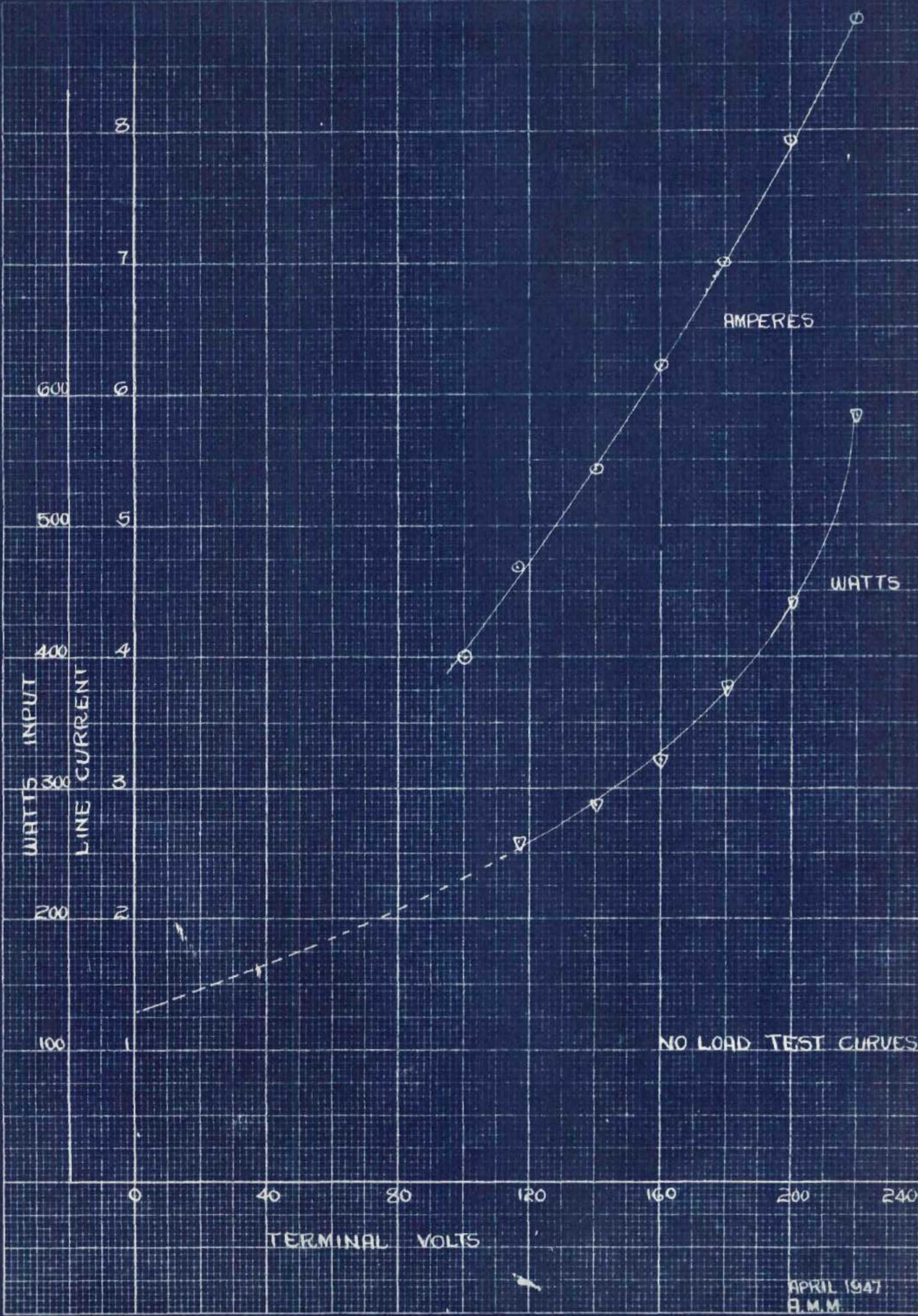
Innumerable other load tests with unbalanced voltages applied to the stator terminals were then made. The unbalanced voltages were obtained by using two voltage regulators connected in open delta, the power coming from the Georgia Power Company lines to insure an infinite bus. This method of unbalancing the voltages by using regulators is far from ideal, but that was the only way available.

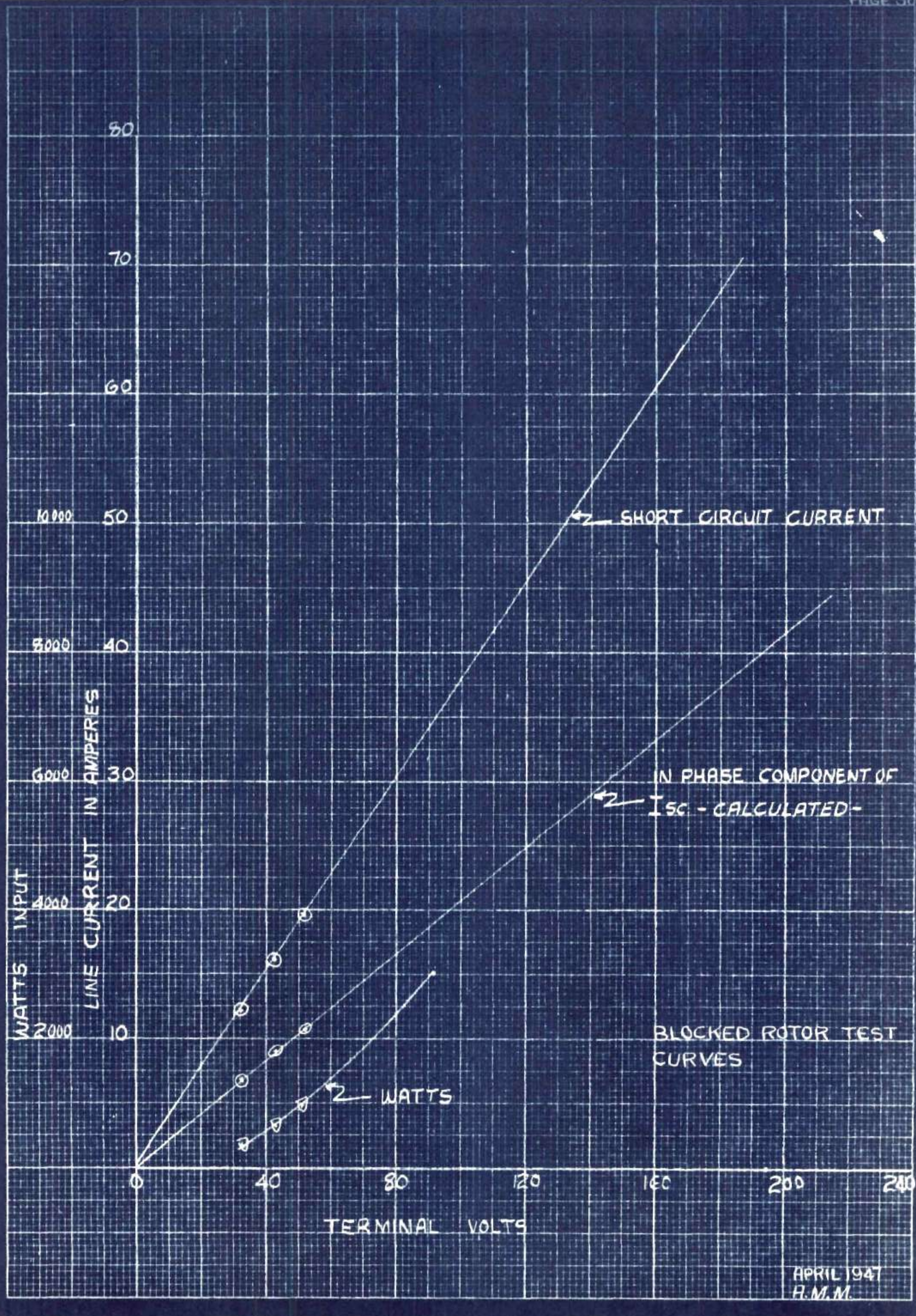
The procedure was to unbalance the voltages to the degree required and then to increase the load until 125% of rated current was read on the high phase. A reading was taken at that point and then the load was decreased in small steps. At each step the voltage on each phase, the current in each phase, power input, speed and torque were read. The same degree of unbalance was maintained throughout the run by means of the voltage regulators. This was not easy to do.

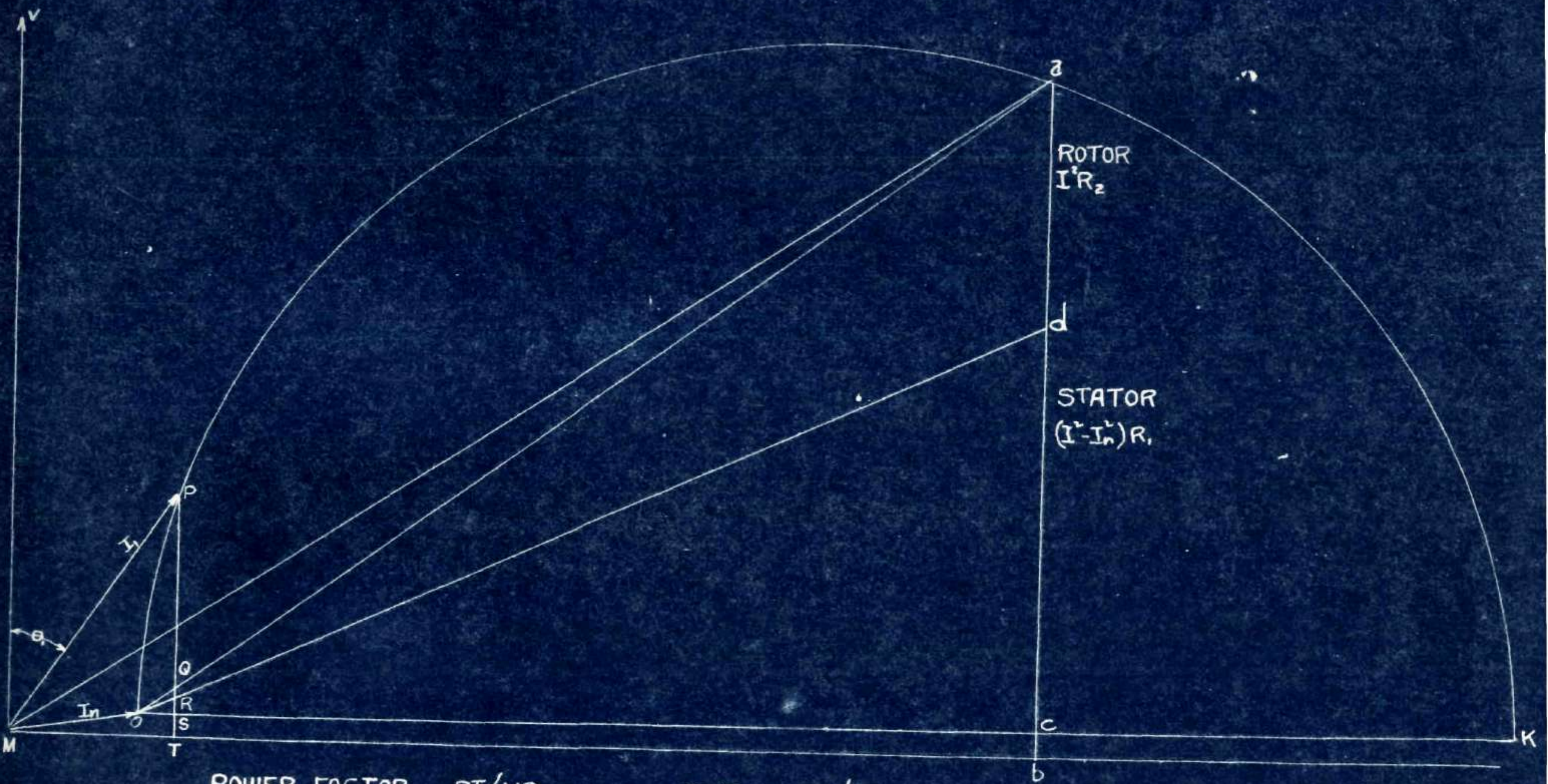
The load test with the greatest degree of voltage unbalance was made when the line voltages were 229 - 200 - 185. The voltages could not be unbalanced beyond that point, because in that case the current in the high phase approached to nearly 125 per cent of rated motor current at practically no load.

Single-Phase Load Test

The motor was started on rated, balanced, three-phase voltages. It was loaded up to approximately full load and then one of the power lines was opened. The motor continued to operate on single phase. The load was gradually decreased and at each point the volts, amperes, watts input, speed, and torque were read.


PAGE 29
WATTS INPUT
LINE CURRENT
100
200
300
400
500
600
1
2
3
4
5
6
7
8
AMPERES
WATTS
NO LOAD TEST CURVES
0
40
80
120
160
200
240
TERMINAL VOLTS
APRIL 1947
A.M.M.


80
70
60
10000
50
8000
40
6000
30
4000
20
2000
10
0
40
80
120
160
200
240
WATTS INPUT
LINE CURRENT IN AMPERES
TERMINAL VOLTS
SHORT CIRCUIT CURRENT
IN PHASE COMPONENT OF
Isc - CALCULATED -
WATTS
BLOCKED ROTOR TEST
CURVES
APRIL 1947
H.M.M.



POWER FACTOR PT/MP

EFFICIENCY QP/TP

POWER OUTPUT QP

SLIP RQ/RP

STATOR I MP

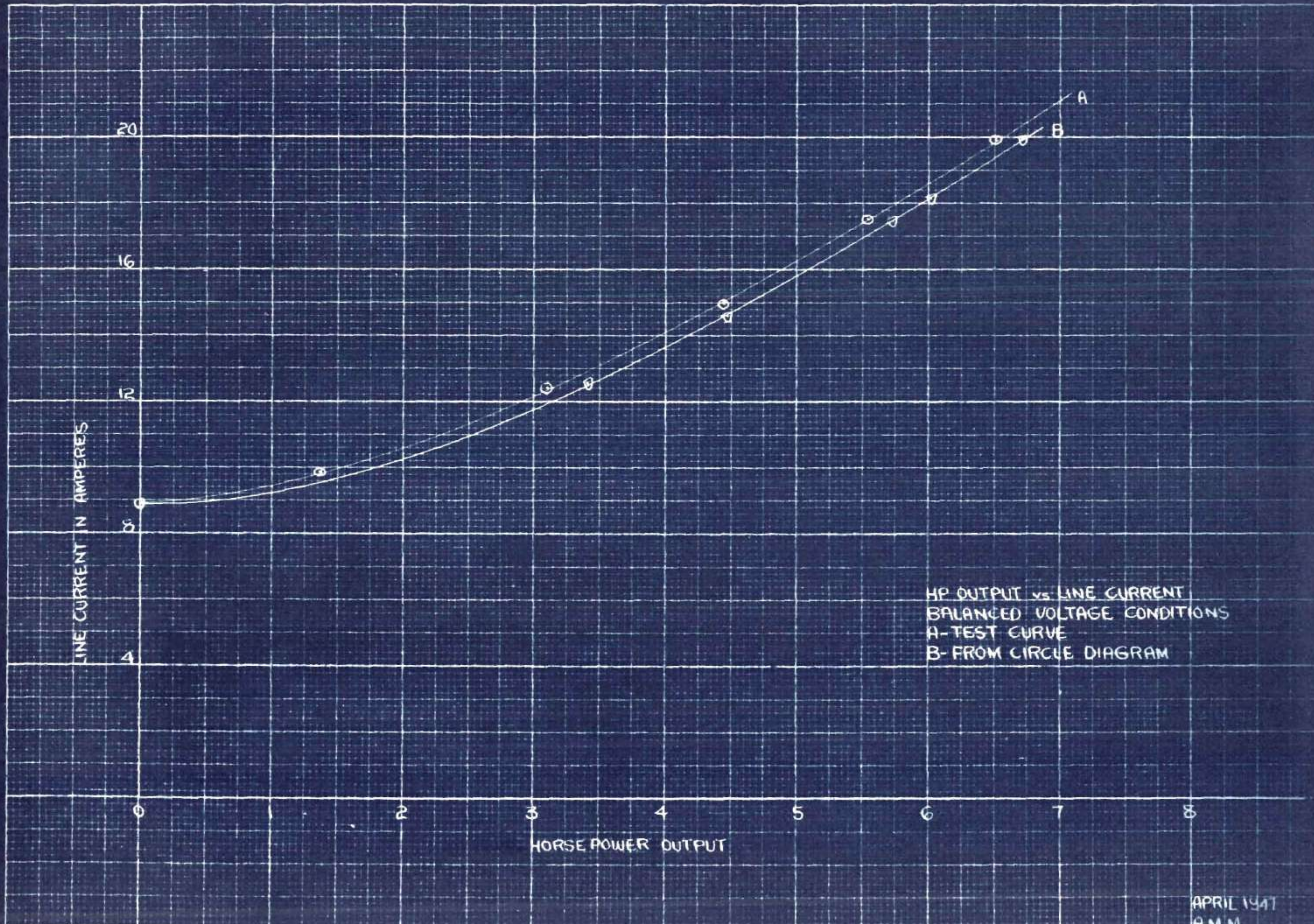
ROTOR I OP

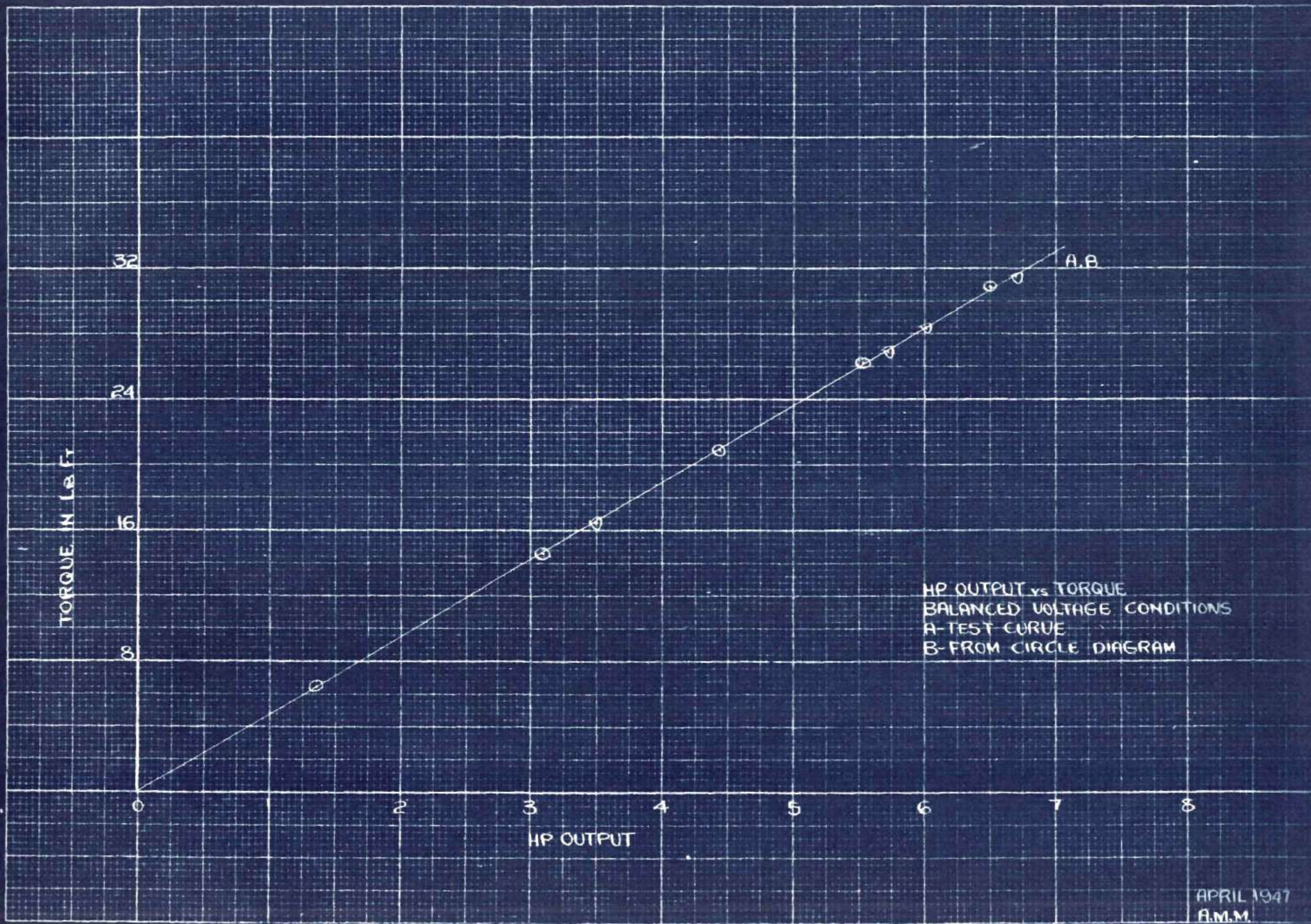
CIRCLE DIAGRAM

SCALE : 1 AMPERE : 1"

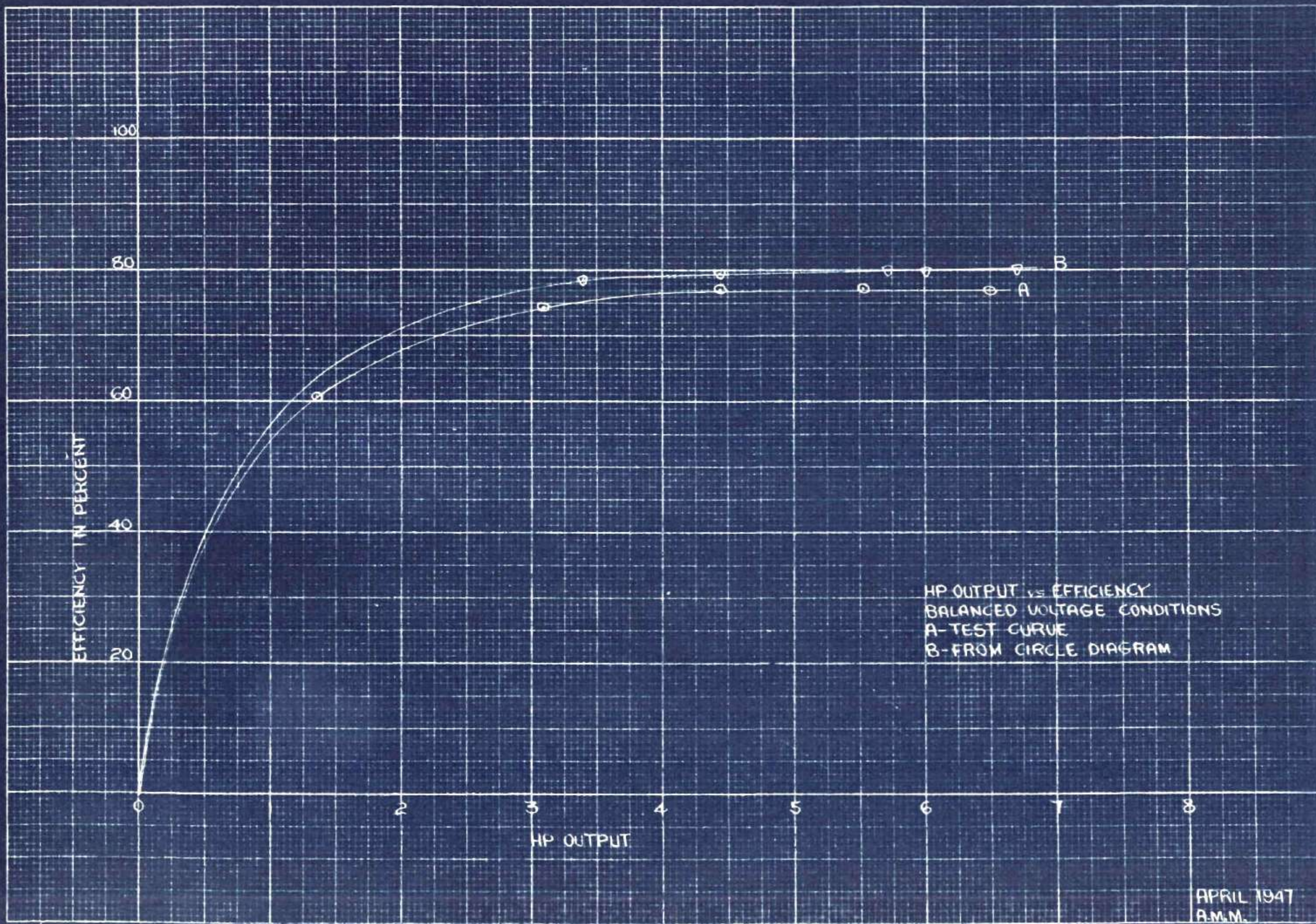
APRIL 1947

A.M.M.

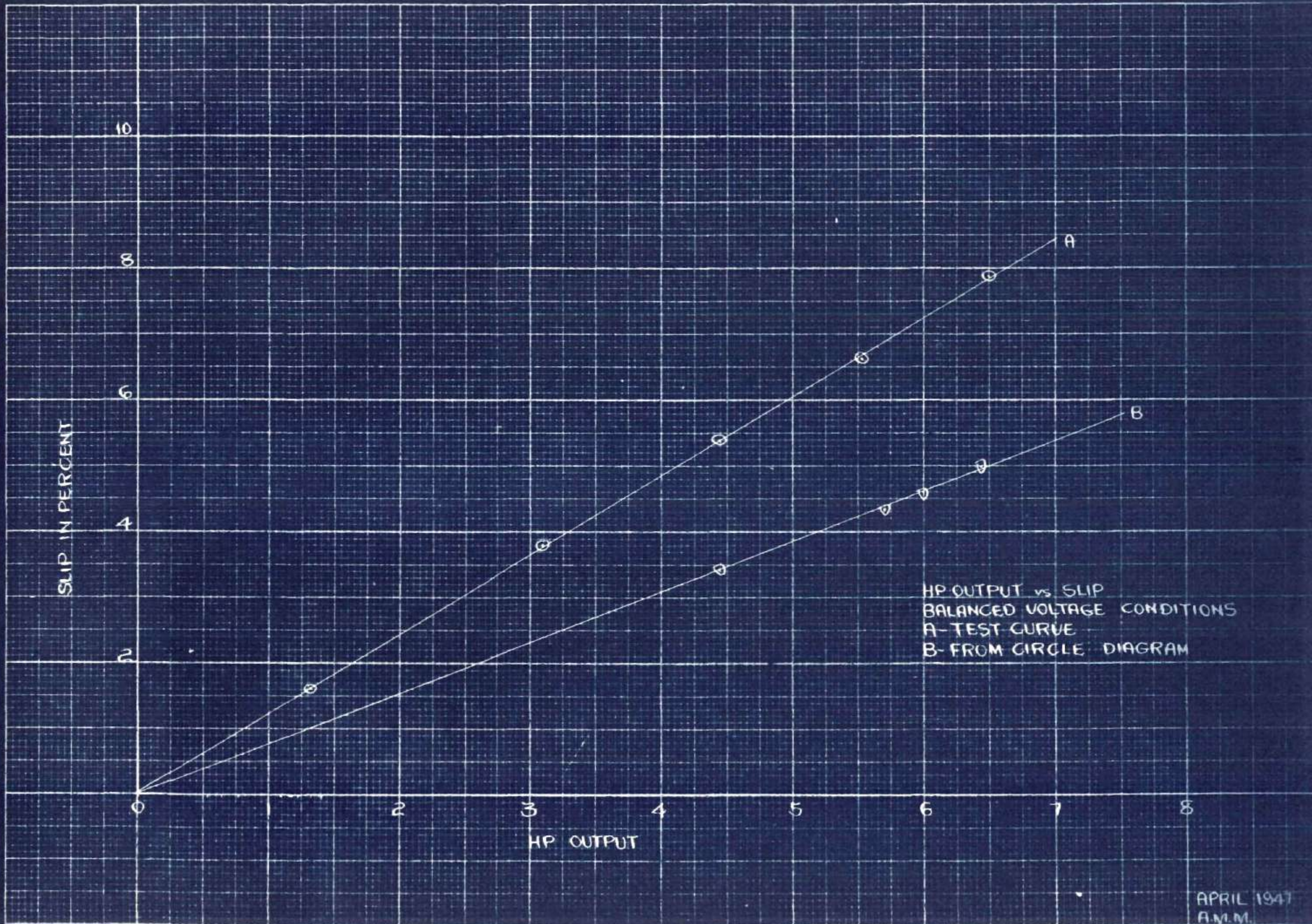

LINE CURRENT IN AMPERES
20
16
12
8
4
0
1
2
3
4
5
6
7
8
HORSE POWER OUTPUT
A
B
HP OUTPUT vs LINE CURRENT
BALANCED VOLTAGE CONDITIONS
A- TEST CURVE
B- FROM CIRCLE DIAGRAM
APRIL 1947
A.M.M

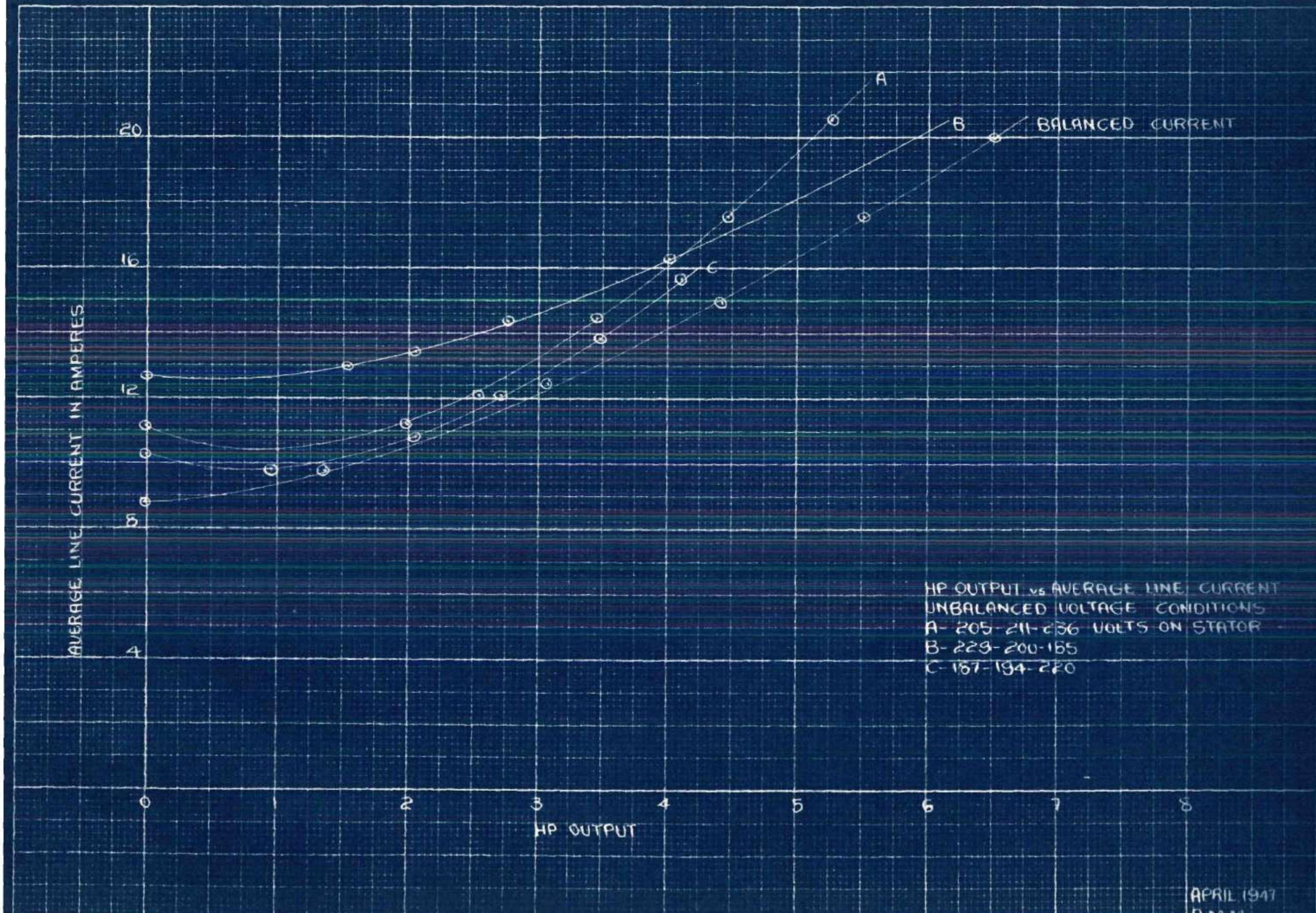


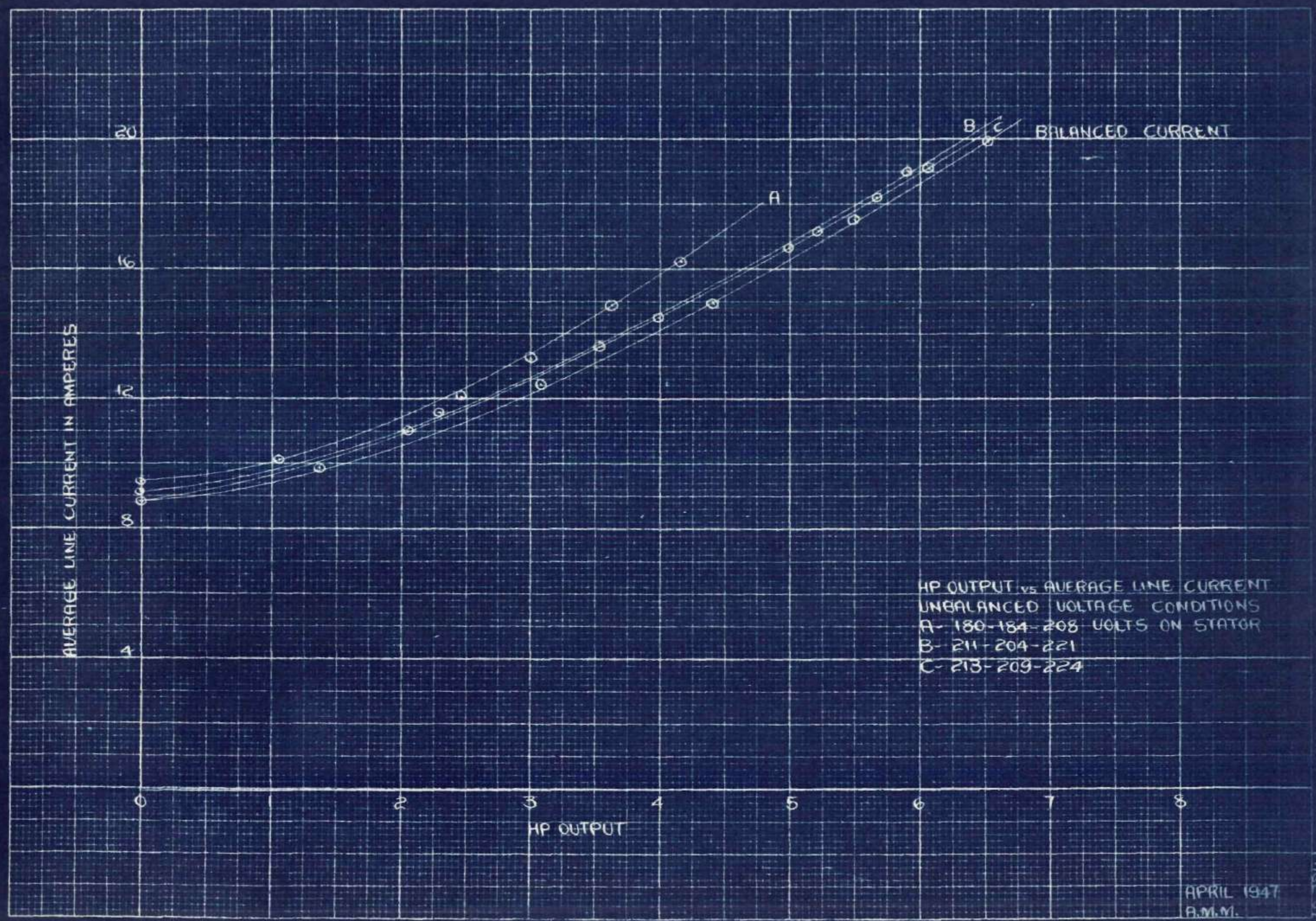
32
24
16
8
TORQUE IN LB FT
0
2
3
4
5
6
7
8
HP OUTPUT
A.B
HP OUTPUT vs TORQUE
BALANCED VOLTAGE CONDITIONS
A-TEST CURVE
B-FROM CIRCLE DIAGRAM
APRIL 1947
A.M.M.



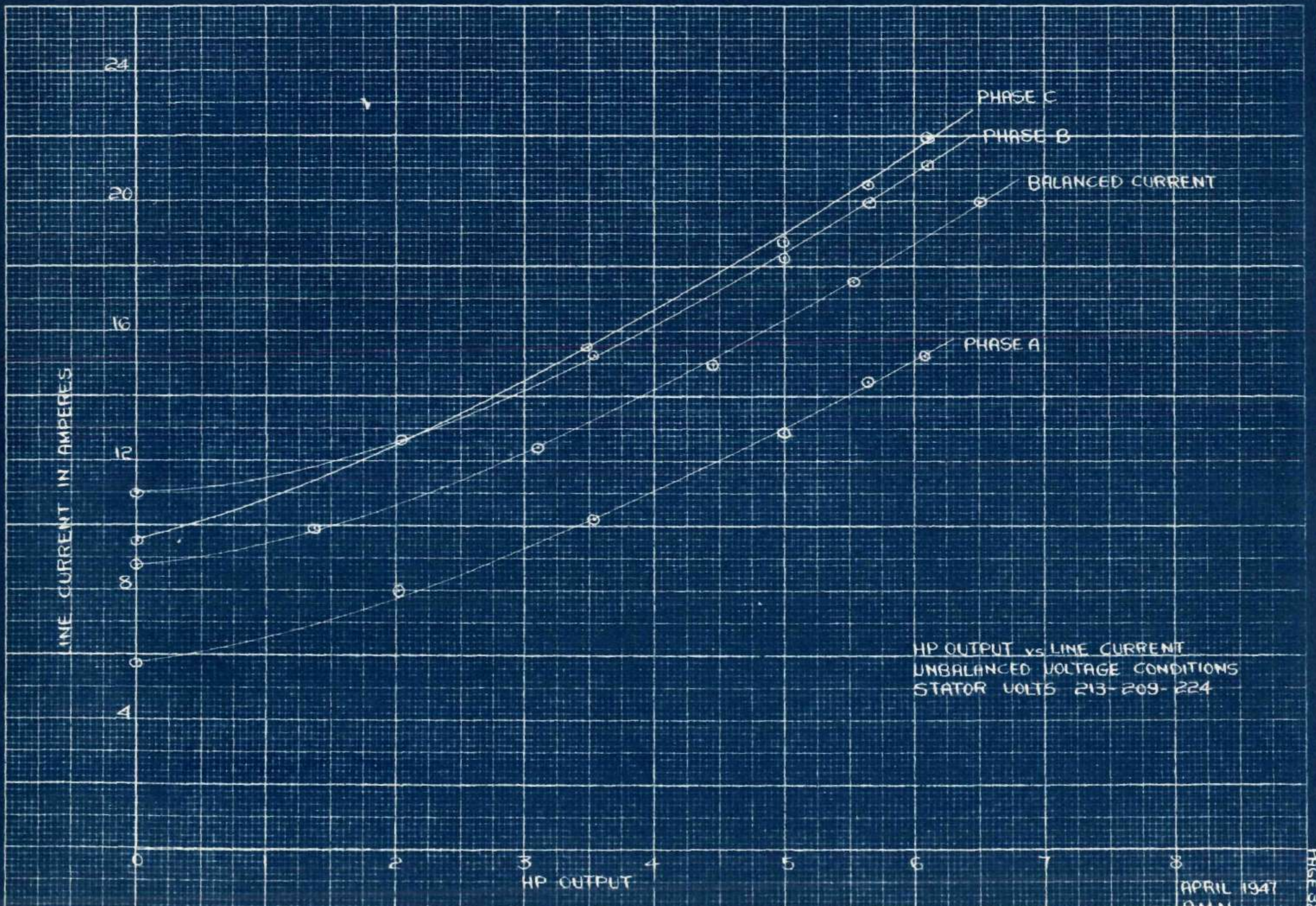
100
80
60
40
20
EFFICIENCY IN PERCENT
0
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP OUTPUT vs EFFICIENCY
BALANCED VOLTAGE CONDITIONS
A-TEST CURVE
B-FROM CIRCLE DIAGRAM
APRIL 1947
A.M.M.

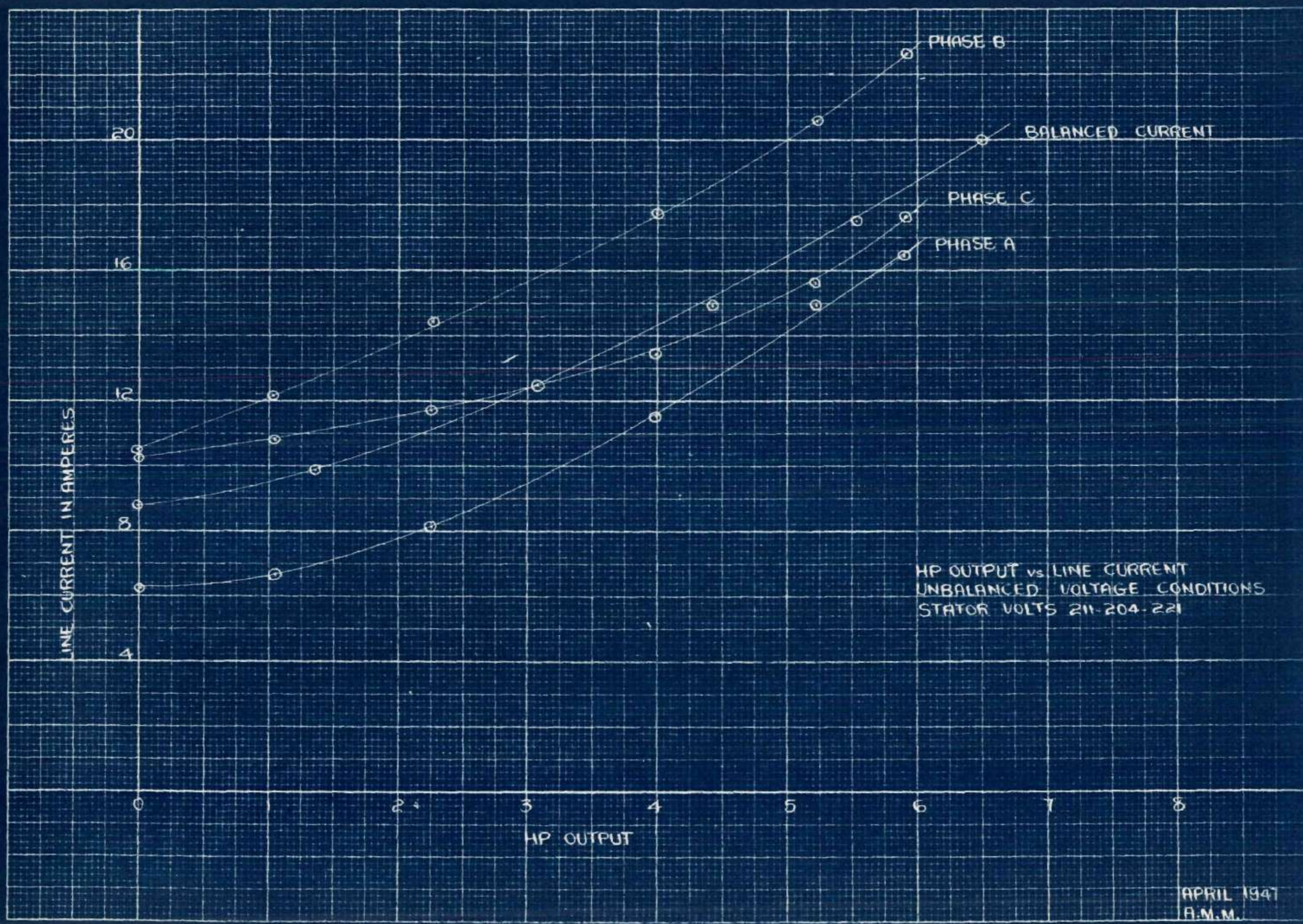

SLIP IN PERCENT
HP OUTPUT
0
1
2
3
4
5
6
7
8
10
A
B
HP OUTPUT vs SLIP
BALANCED VOLTAGE CONDITIONS
A- TEST CURVE
B- FROM CIRCLE DIAGRAM
APRIL 1947
A.M.M.

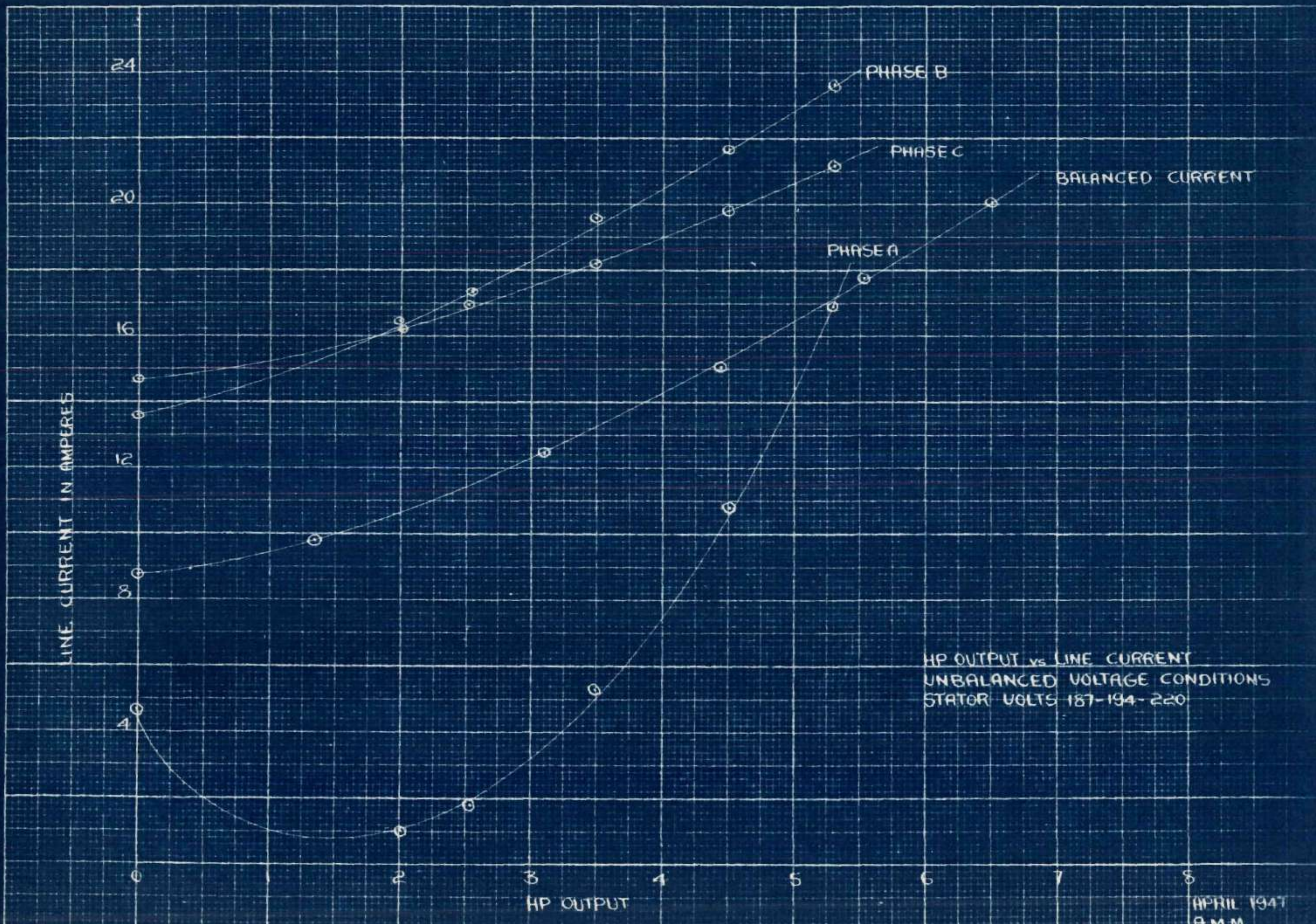

A
B
BALANCED CURRENT
C
20
16
12
8
4
AVERAGE LINE CURRENT IN AMPERES
0
1
2
3
4
5
6
7
8
HP OUTPUT
HP OUTPUT vs AVERAGE LINE CURRENT
UNBALANCED VOLTAGE CONDITIONS
A- 205-211-236 VOLTS ON STATOR
B- 229-200-165
C- 187-194-220
APRIL 1947
A.M.M.

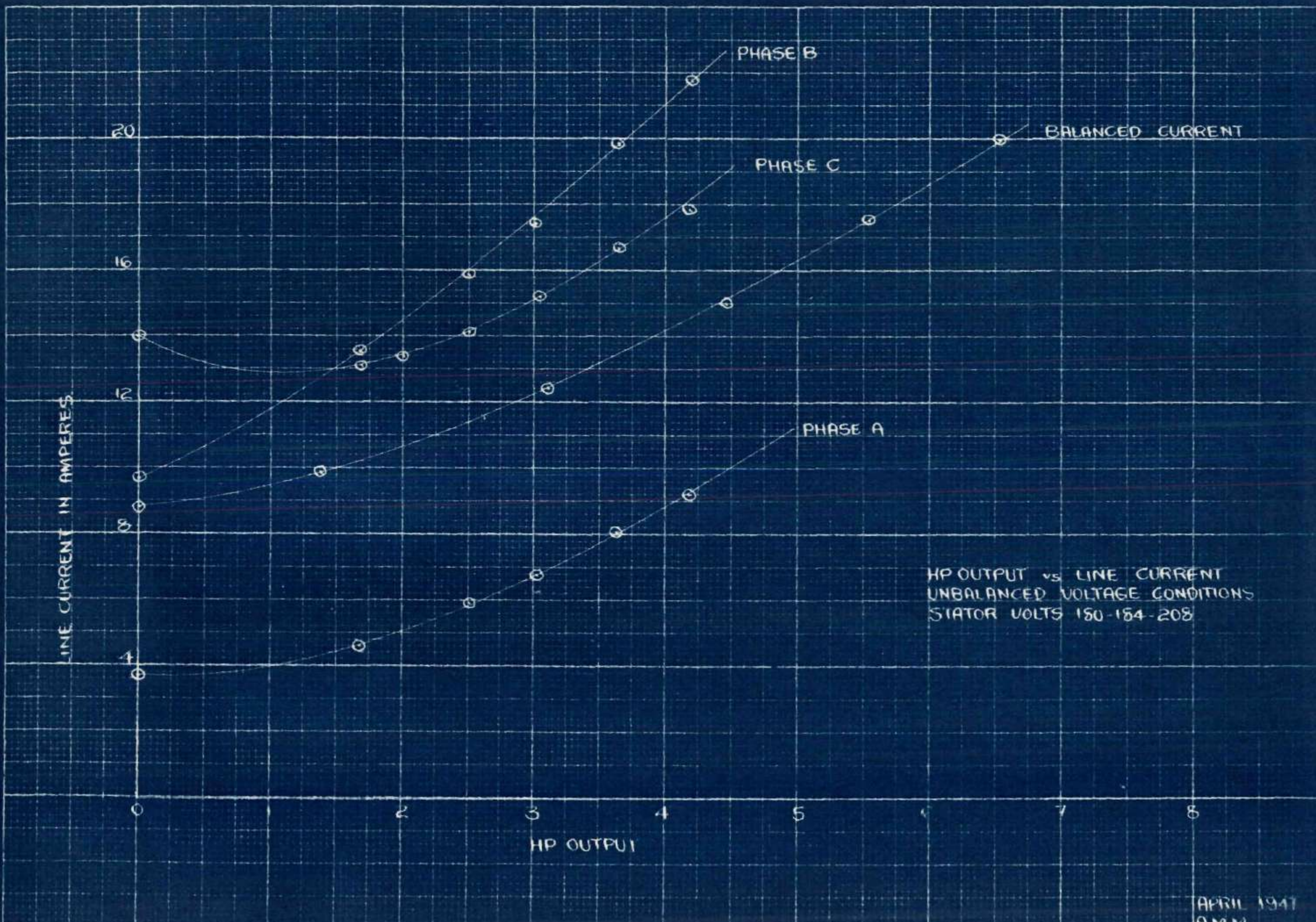


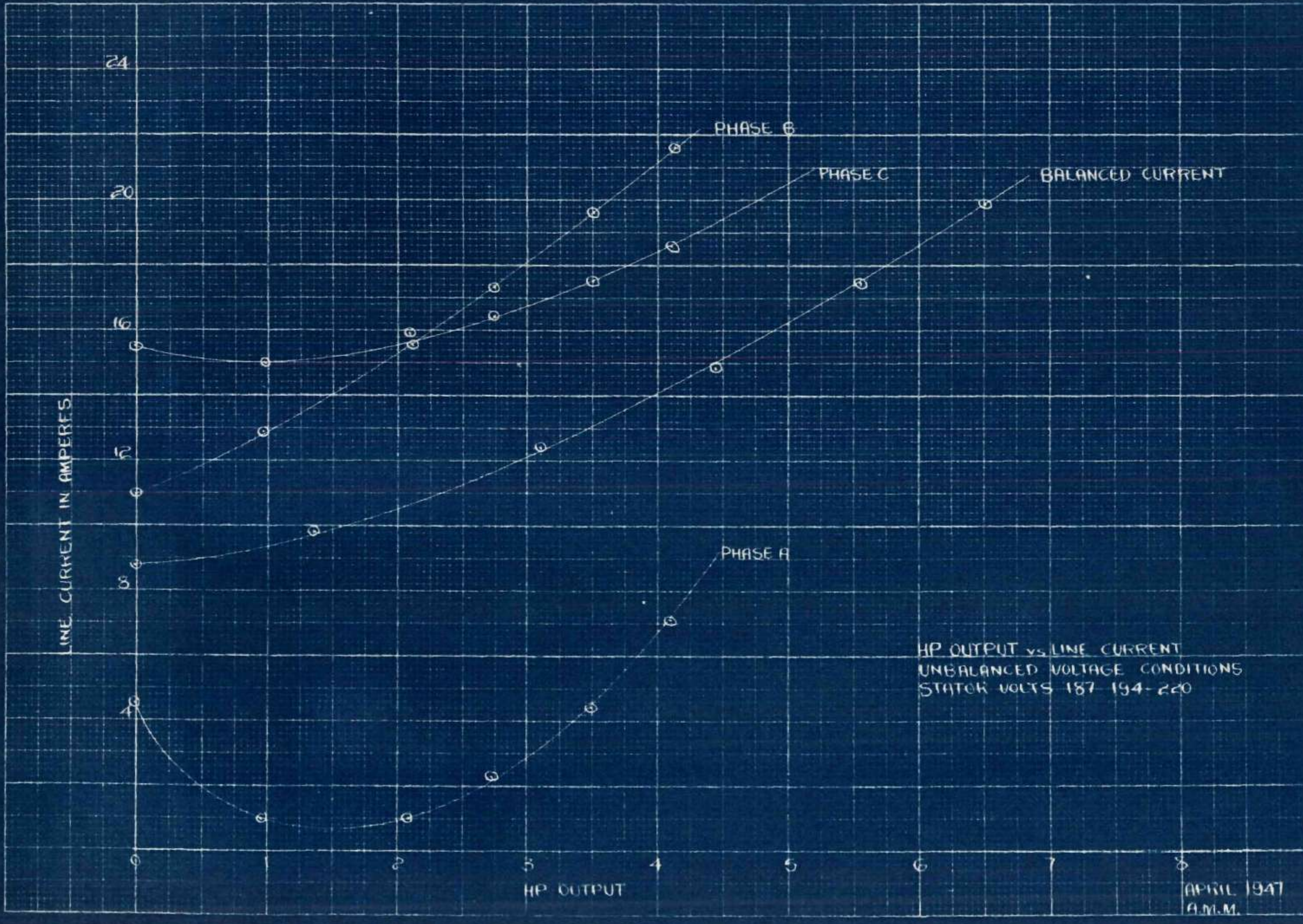
20
16
12
8
4
AVERAGE LINE CURRENT IN AMPERES
0
2
3
4
5
6
7
8
HP OUTPUT
A
B
C
BALANCED CURRENT
HP OUTPUT vs AVERAGE LINE CURRENT
UNBALANCED VOLTAGE CONDITIONS
A- 180-184-208 VOLTS ON STATOR
B- 211-204-221
C- 213-209-224
APRIL 1947
A.M.M.
PAGE 37


24
20
16
12
8
4
LINE CURRENT IN AMPERES
PHASE C
PHASE B
BALANCED CURRENT
PHASE A
0
2
3
4
5
6
7
8
HP OUTPUT
HP OUTPUT vs LINE CURRENT
UNBALANCED VOLTAGE CONDITIONS
STATOR VOLTS 213-209-224
APRIL 1947
A.M.N.

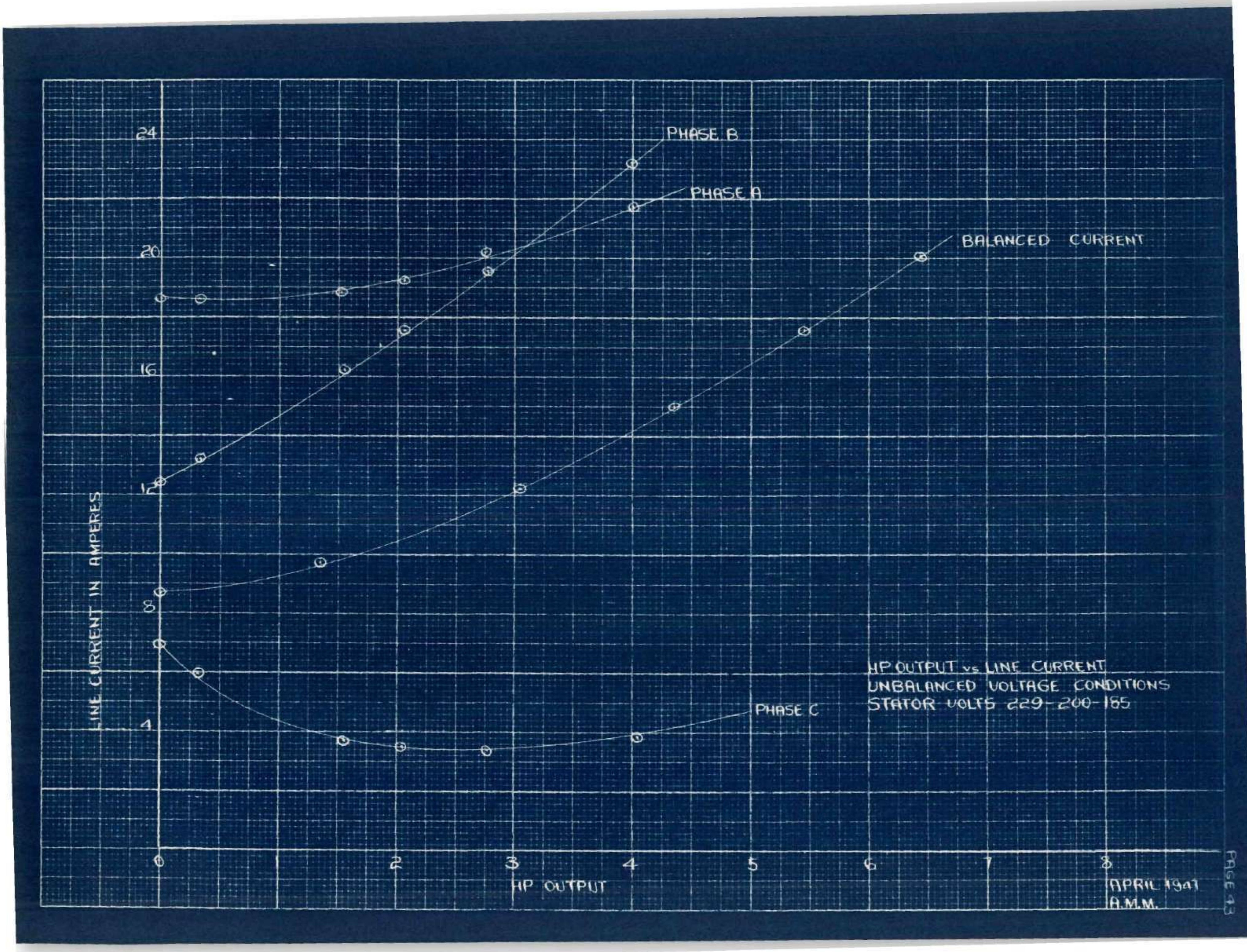

PHASE B
BALANCED CURRENT
PHASE C
PHASE A
20
16
12
8
4
LINE CURRENT IN AMPERES
0
2
3
4
5
6
7
8
HP OUTPUT
HP OUTPUT vs LINE CURRENT
UNBALANCED VOLTAGE CONDITIONS
STATOR VOLTS 211-204-221
APRIL 1947
A.M.M.

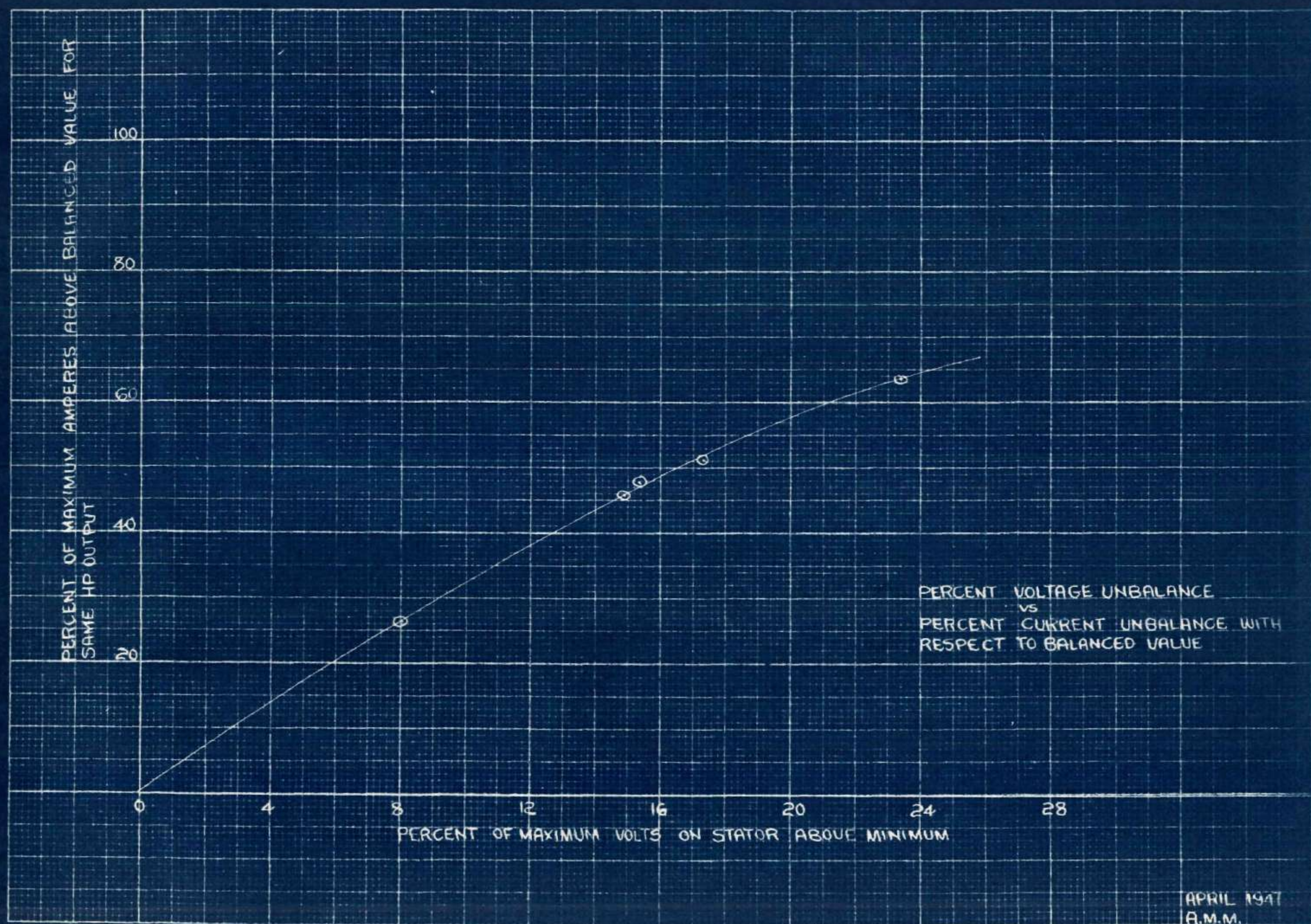

PHASE B
PHASE C
BALANCED CURRENT
PHASE A
LINE CURRENT IN AMPERES
24
20
16
12
8
4
0
1
2
3
4
5
6
7
8
HP OUTPUT
HP OUTPUT vs LINE CURRENT
UNBALANCED VOLTAGE CONDITIONS
STATOR VOLTS 187-194-220
APRIL 1947
A.M.M.


PHASE B
BALANCED CURRENT
PHASE C
PHASE A
LINE CURRENT IN AMPERES
20
16
12
8
4
0
1
2
3
4
5
6
7
8
HP OUTPUT
HP OUTPUT vs LINE CURRENT
UNBALANCED VOLTAGE CONDITIONS
STATOR VOLTS 180-184-208
APRIL 1941
A.M.M.

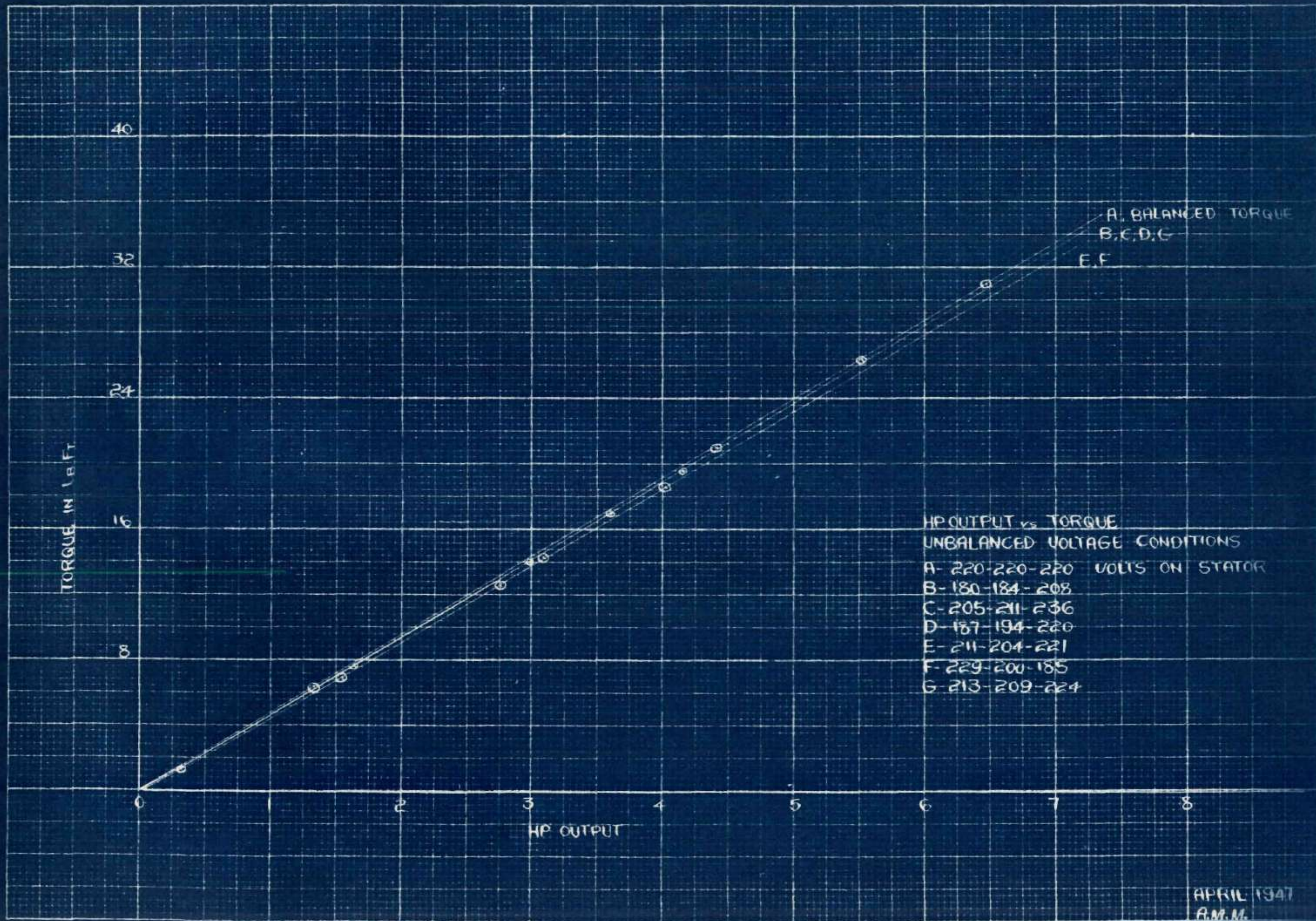


PHASE B
PHASE C
BALANCED CURRENT
PHASE A
24
20
16
12
8
4
0
1
2
3
4
5
6
7
8
LINE CURRENT IN AMPERES
HP OUTPUT
HP OUTPUT vs LINE CURRENT
UNBALANCED VOLTAGE CONDITIONS
STATOR VOLTS 187-194-220
APRIL 1947
A.M.M.

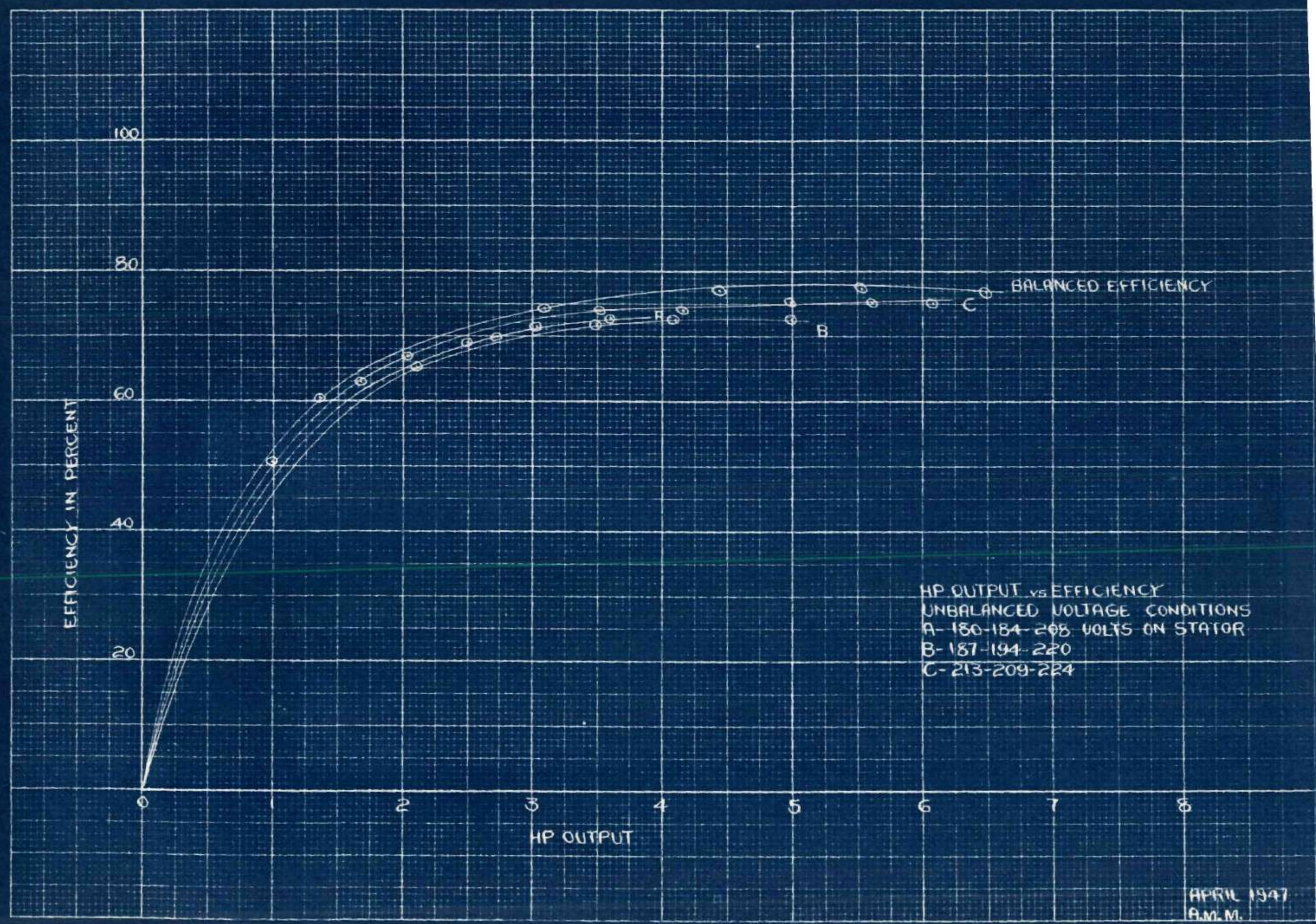

PHASE B
PHASE A
BALANCED CURRENT
PHASE C
LINE CURRENT IN AMPERES
HP OUTPUT
24
20
16
12
8
4
0
1
2
3
4
5
6
7
8
HP OUTPUT vs LINE CURRENT
UNBALANCED VOLTAGE CONDITIONS
STATOR VOLTS 229-200-185
APRIL 1947
A.M.M.
PAGE 43



PERCENT OF MAXIMUM AMPERES ABOVE BALANCED VALUE FOR SAME HP OUTPUT
100
80
60
40
20
0
4
8
12
16
20
24
28
PERCENT OF MAXIMUM VOLTS ON STATOR ABOVE MINIMUM
PERCENT VOLTAGE UNBALANCE
VS
PERCENT CURRENT UNBALANCE WITH RESPECT TO BALANCED VALUE
APRIL 1947
A.M.M.



A. BALANCED TORQUE
B, C, D, G
E, F
40
32
24
16
8
0
1
2
3
4
5
6
7
8
TORQUE IN LB.FT.
HP OUTPUT
HP OUTPUT vs TORQUE
UNBALANCED VOLTAGE CONDITIONS
A- 220-220-220 VOLTS ON STATOR
B- 180-184-208
C- 205-211-236
D- 187-194-220
E- 211-204-221
F- 229-200-185
G- 213-209-224
APRIL 1947
A.M.M.



EFFICIENCY IN PERCENT
100
80
60
40
20
0
1
2
3
4
5
6
7
8
HP OUTPUT
BALANCED EFFICIENCY
A
B
C
HP OUTPUT vs EFFICIENCY
UNBALANCED VOLTAGE CONDITIONS
A- 180-184-208 VOLTS ON STATOR
B- 187-194-220
C- 213-209-224
APRIL 1947
A.M.M.
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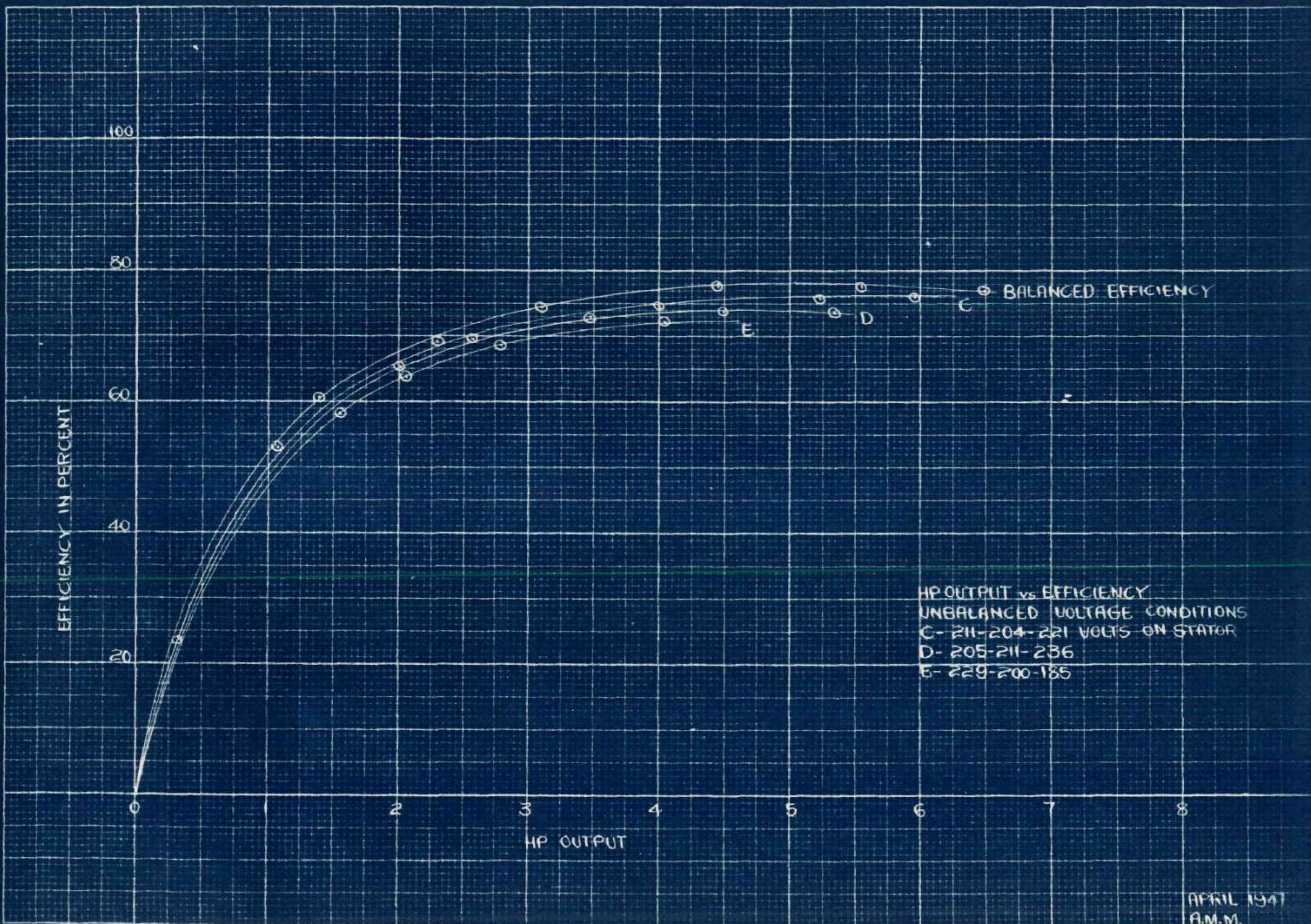
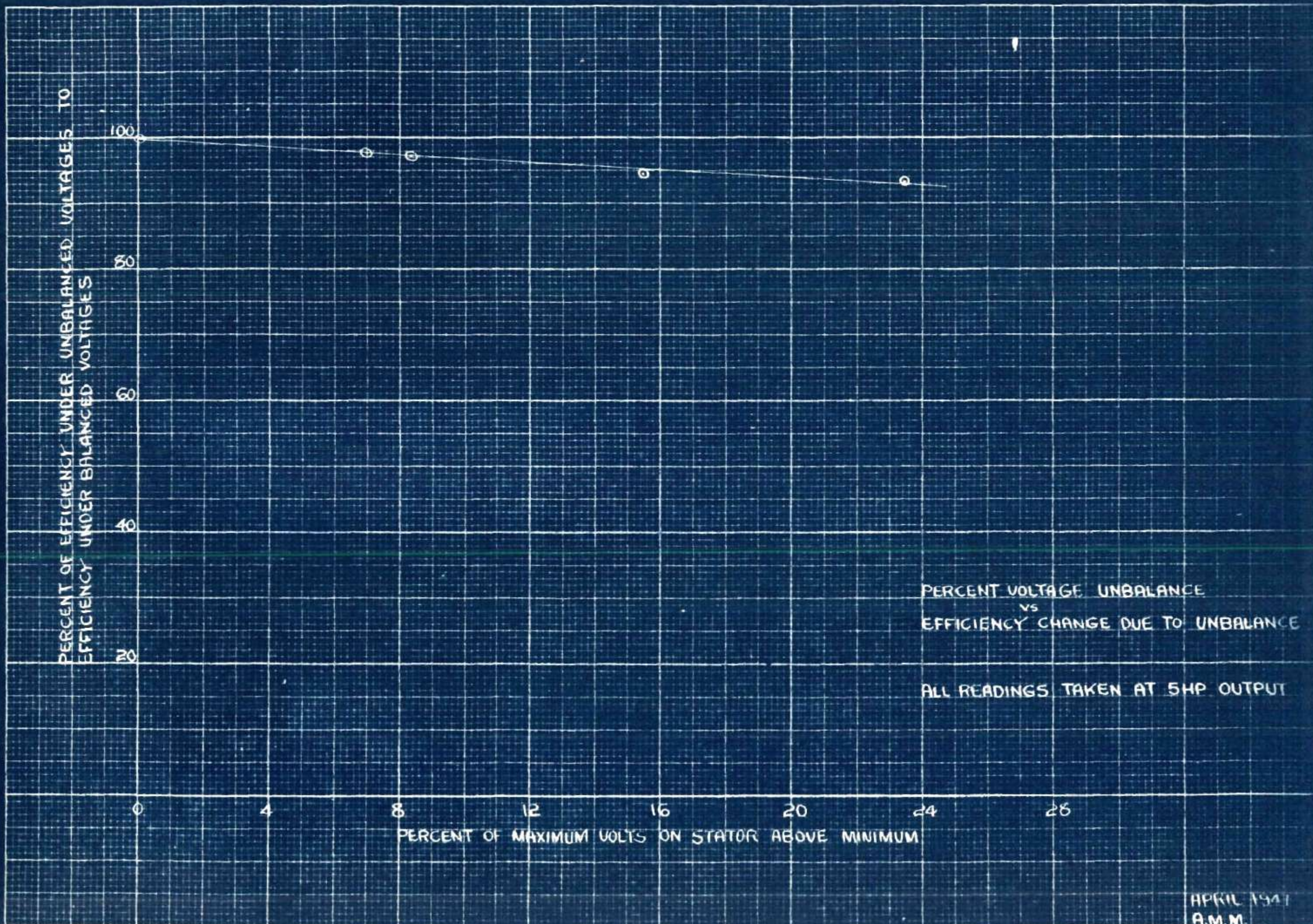
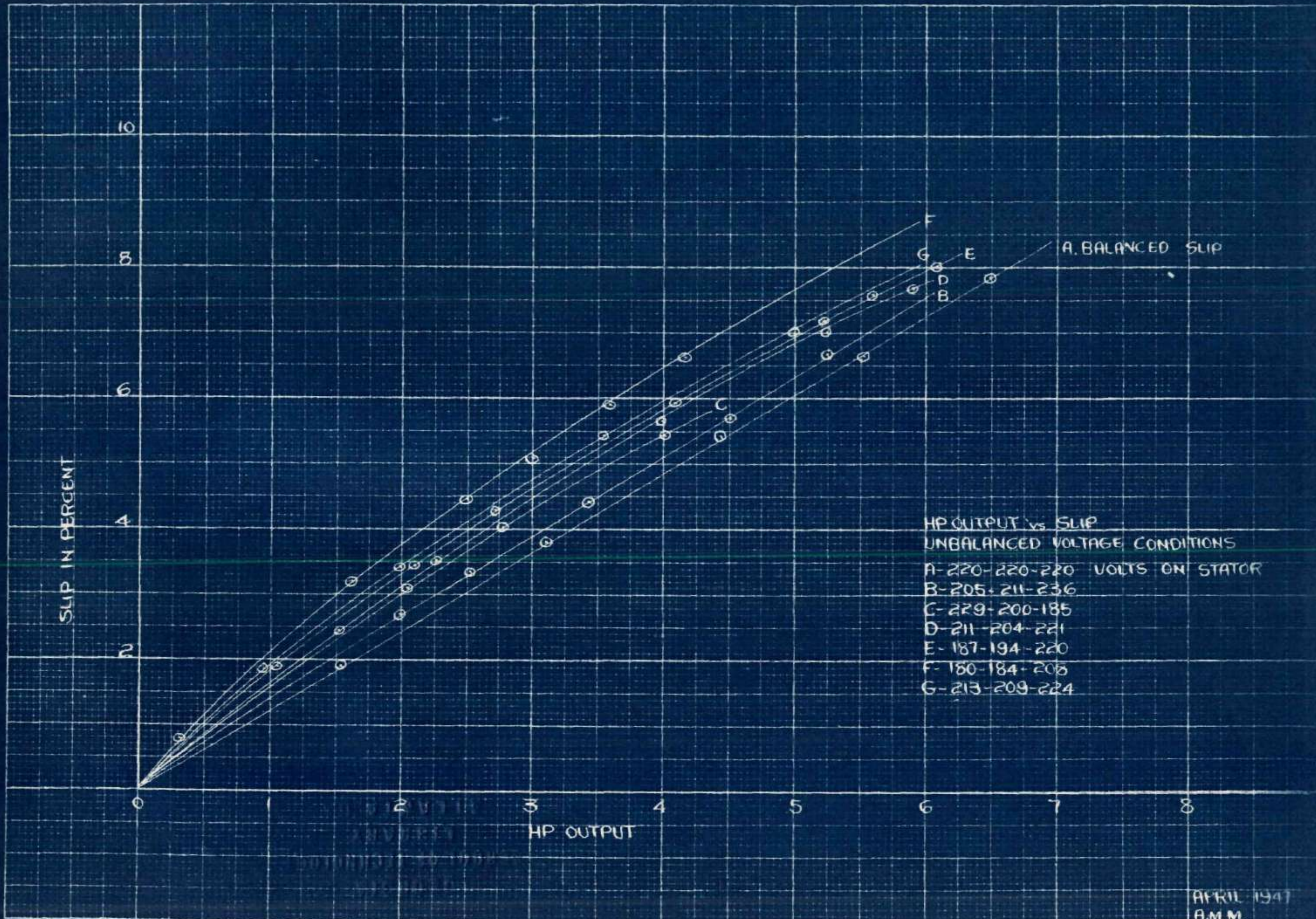
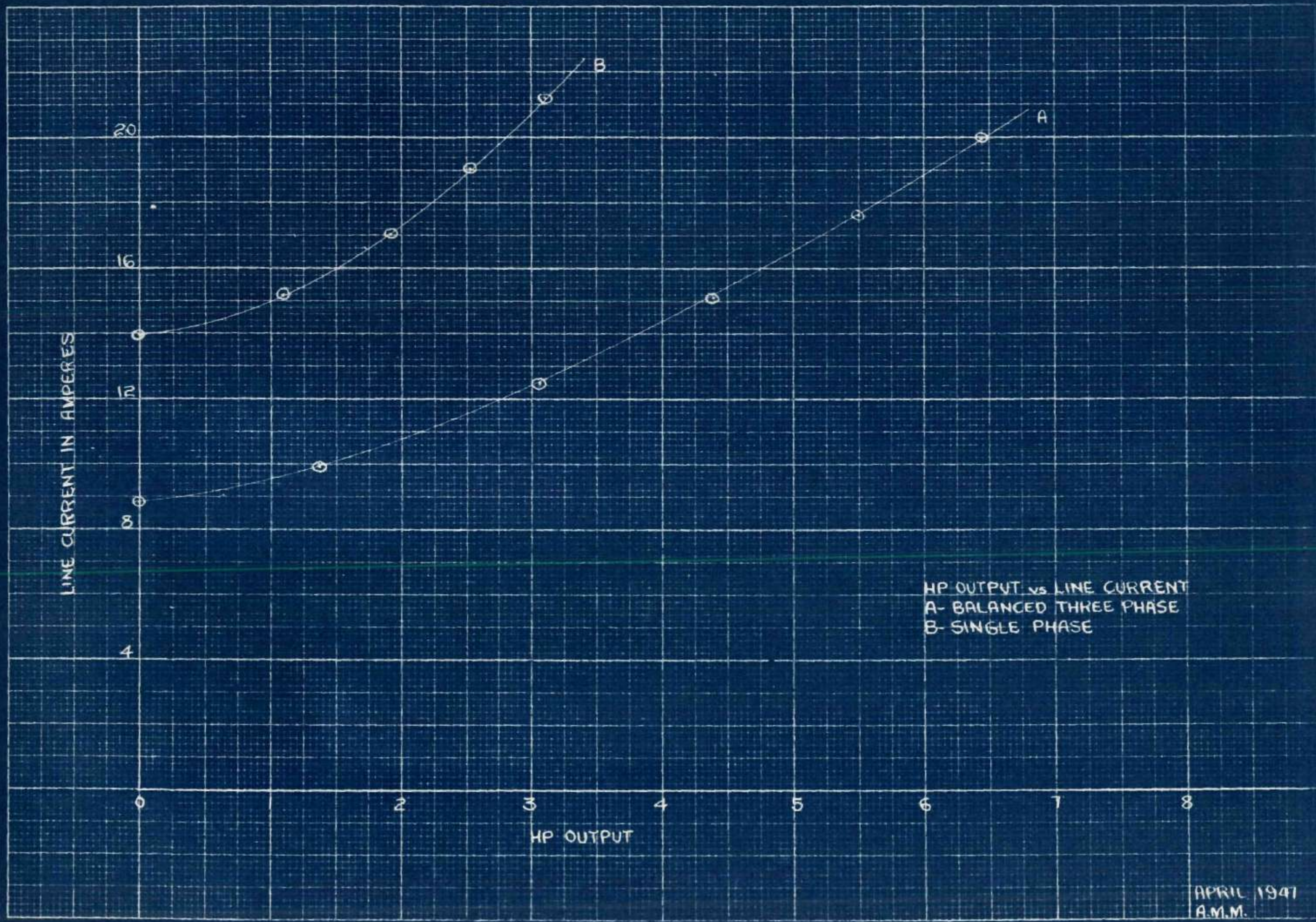
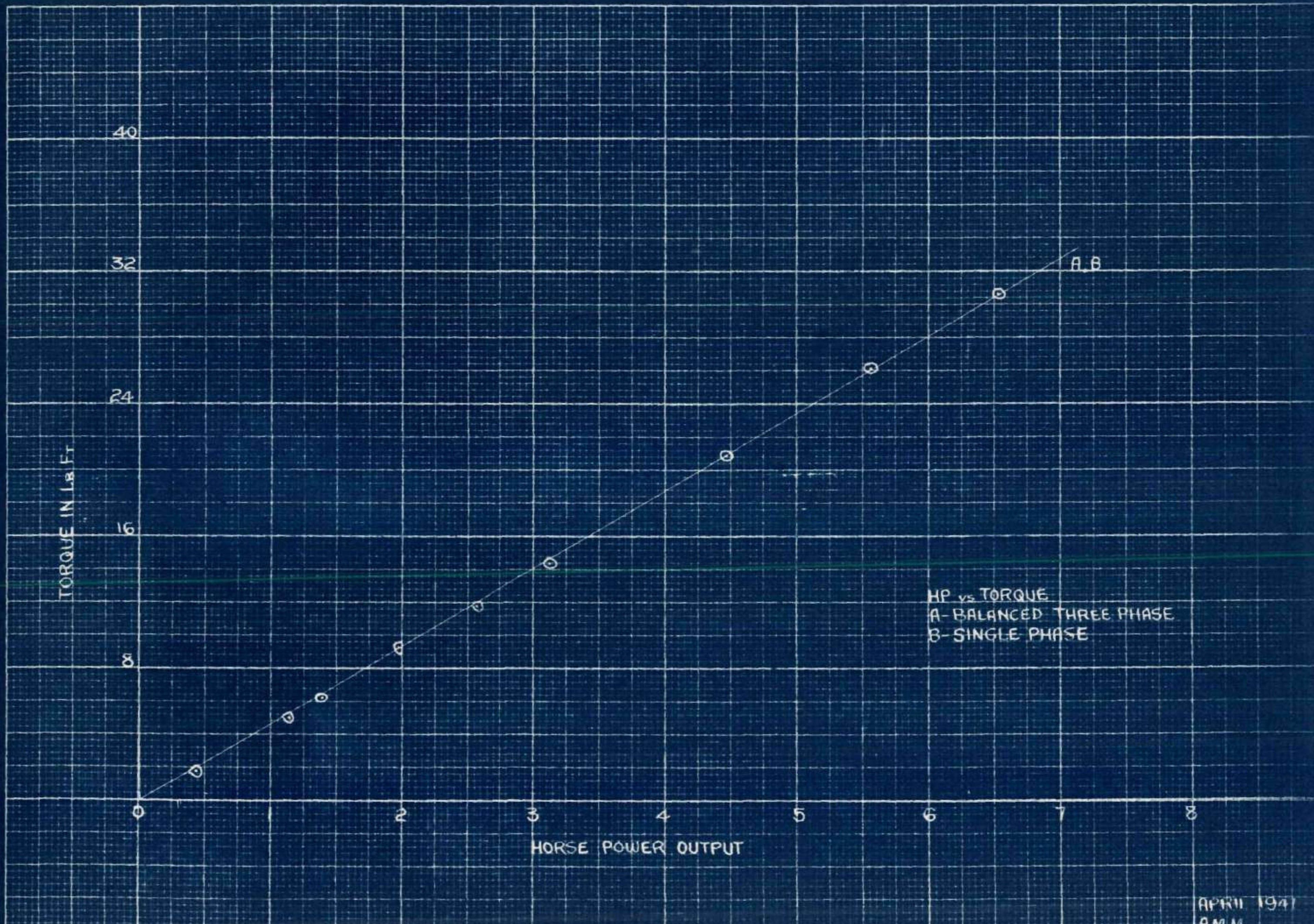

BALANCED EFFICIENCY
C
D
E
100
80
60
40
20
EFFICIENCY IN PERCENT
0
1
2
3
4
5
6
7
8
HP OUTPUT
HP OUTPUT vs EFFICIENCY
UNBALANCED VOLTAGE CONDITIONS
C- 211-204-221 VOLTS ON STATOR
D- 205-211-236
E- 229-200-185
APRIL 1947
A.M.M.

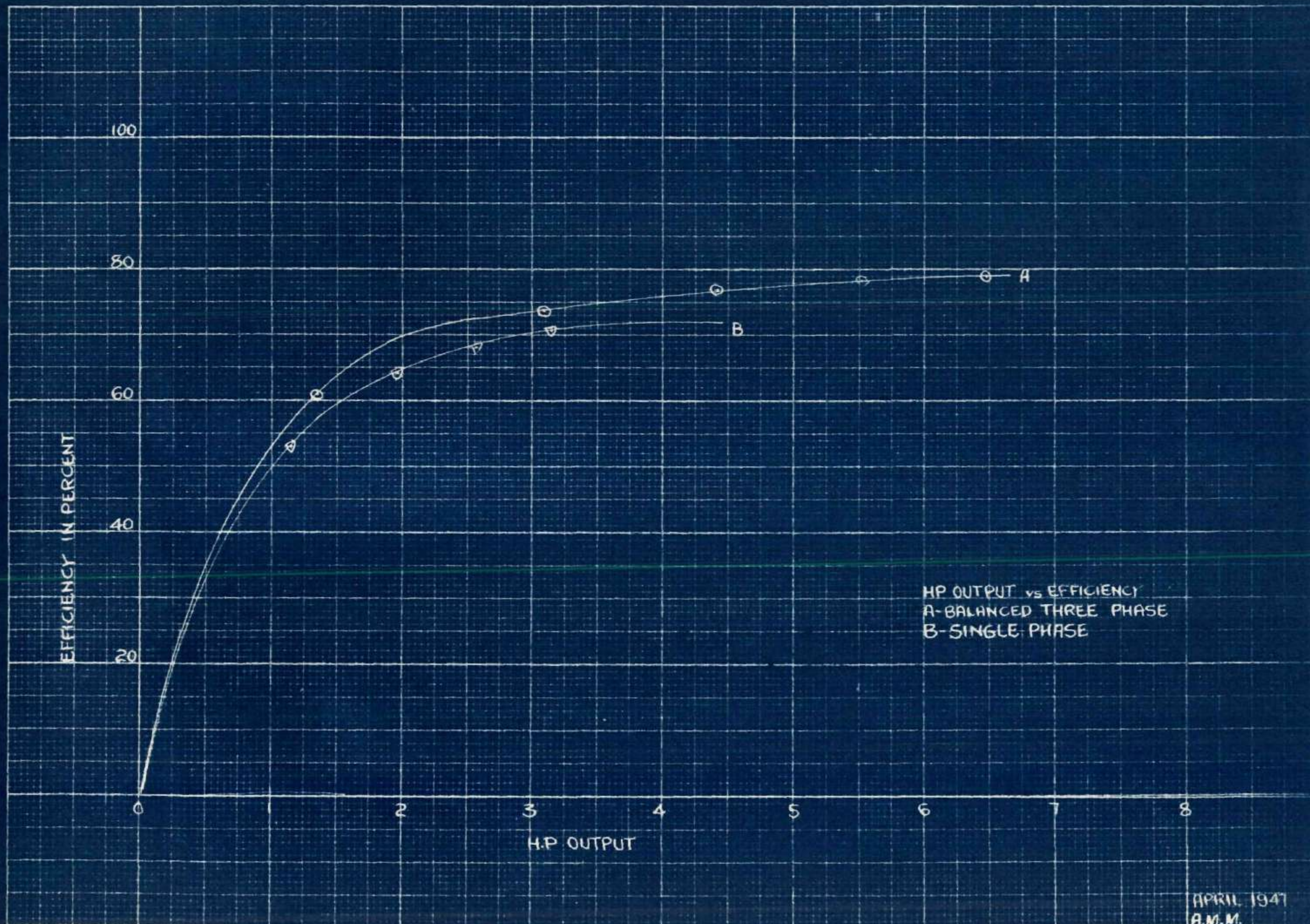

PERCENT OF EFFICIENCY UNDER UNBALANCED VOLTAGES TO EFFICIENCY UNDER BALANCED VOLTAGES
100
80
60
40
20
0
4
8
12
16
20
24
28
PERCENT OF MAXIMUM VOLTS ON STATOR ABOVE MINIMUM
PERCENT VOLTAGE UNBALANCE
VS
EFFICIENCY CHANGE DUE TO UNBALANCE
ALL READINGS TAKEN AT 5HP OUTPUT
APRIL 1947
A.M.M.

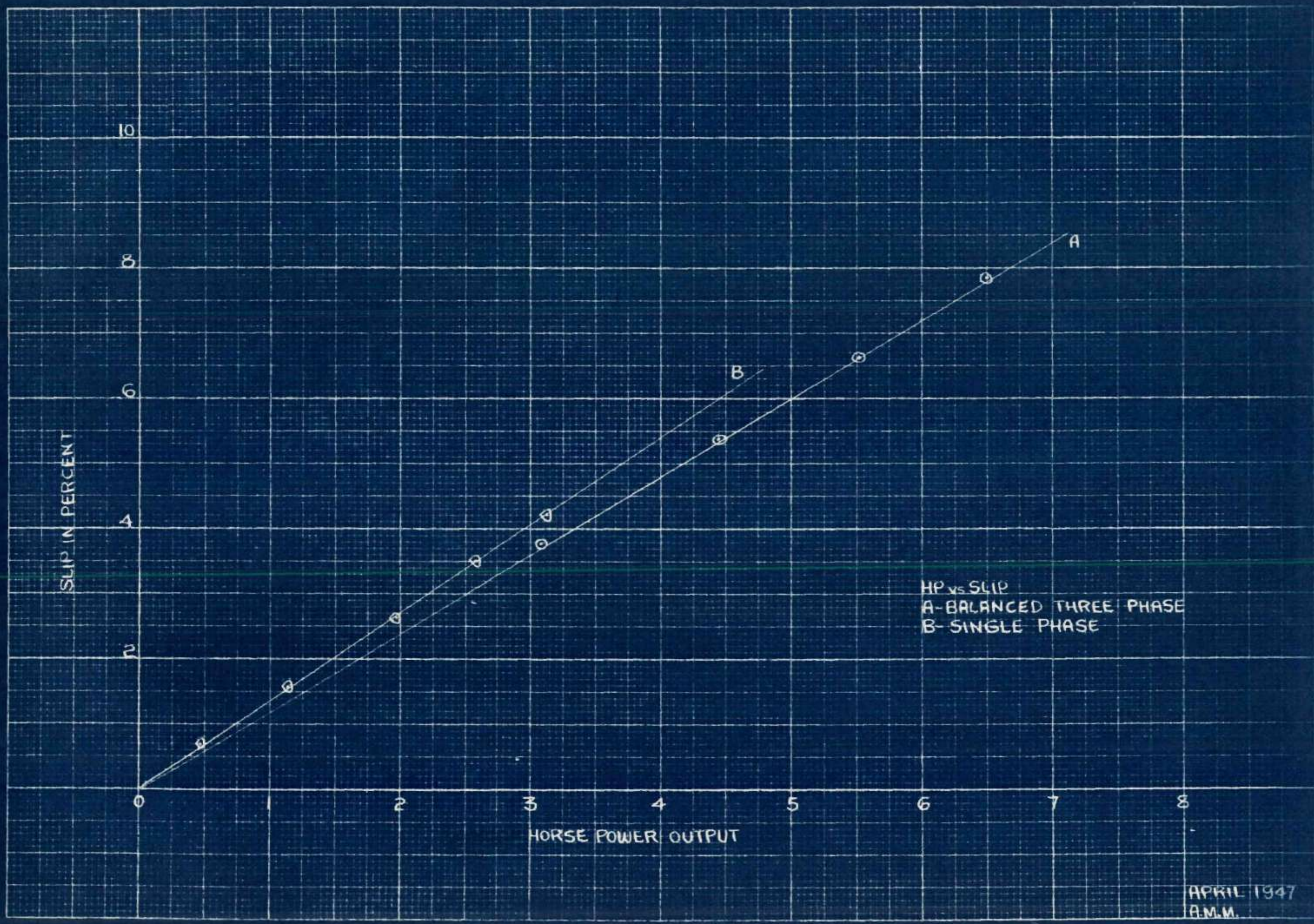

SLIP IN PERCENT
HP OUTPUT
0
1
2
3
4
5
6
7
8
10
F
G
E
D
B
C
A. BALANCED SLIP
HP OUTPUT vs SLIP
UNBALANCED VOLTAGE CONDITIONS
A-220-220-220 VOLTS ON STATOR
B-205-211-236
C-229-200-185
D-211-204-221
E-187-194-220
F-180-184-208
G-213-209-224
APRIL 1947
A.M.M.


B
A
20
16
12
8
4
LINE CURRENT IN AMPERES
0
2
3
4
5
6
7
8
HP OUTPUT
HP OUTPUT vs LINE CURRENT
A- BALANCED THREE PHASE
B- SINGLE PHASE
APRIL 1947
A.M.M.

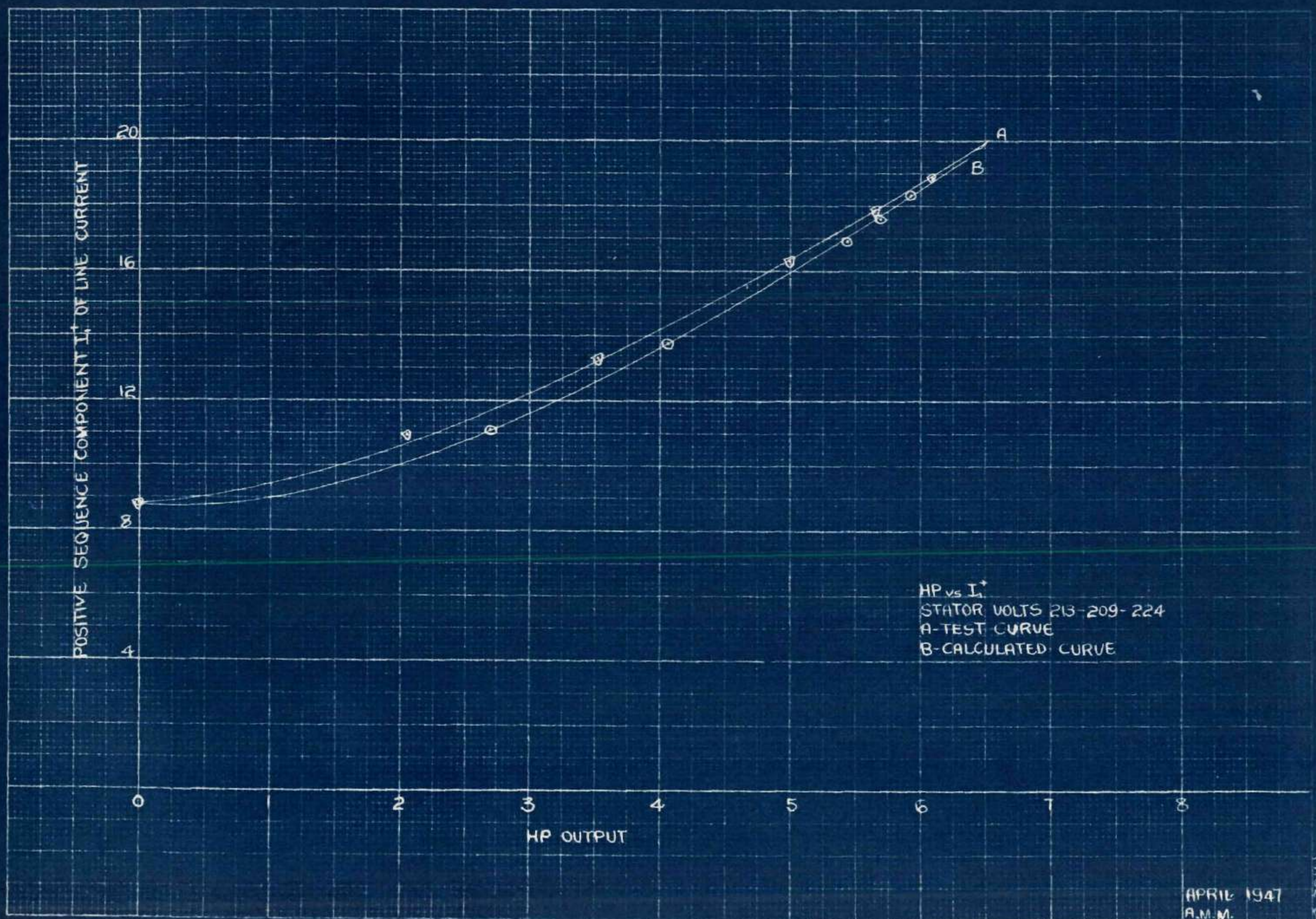


A,B
40
32
24
16
8
TORQUE IN LB FT
0
1
2
3
4
5
6
7
8
HORSE POWER OUTPUT
HP vs TORQUE
A- BALANCED THREE PHASE
B- SINGLE PHASE
APRIL 1941
A.M.M.

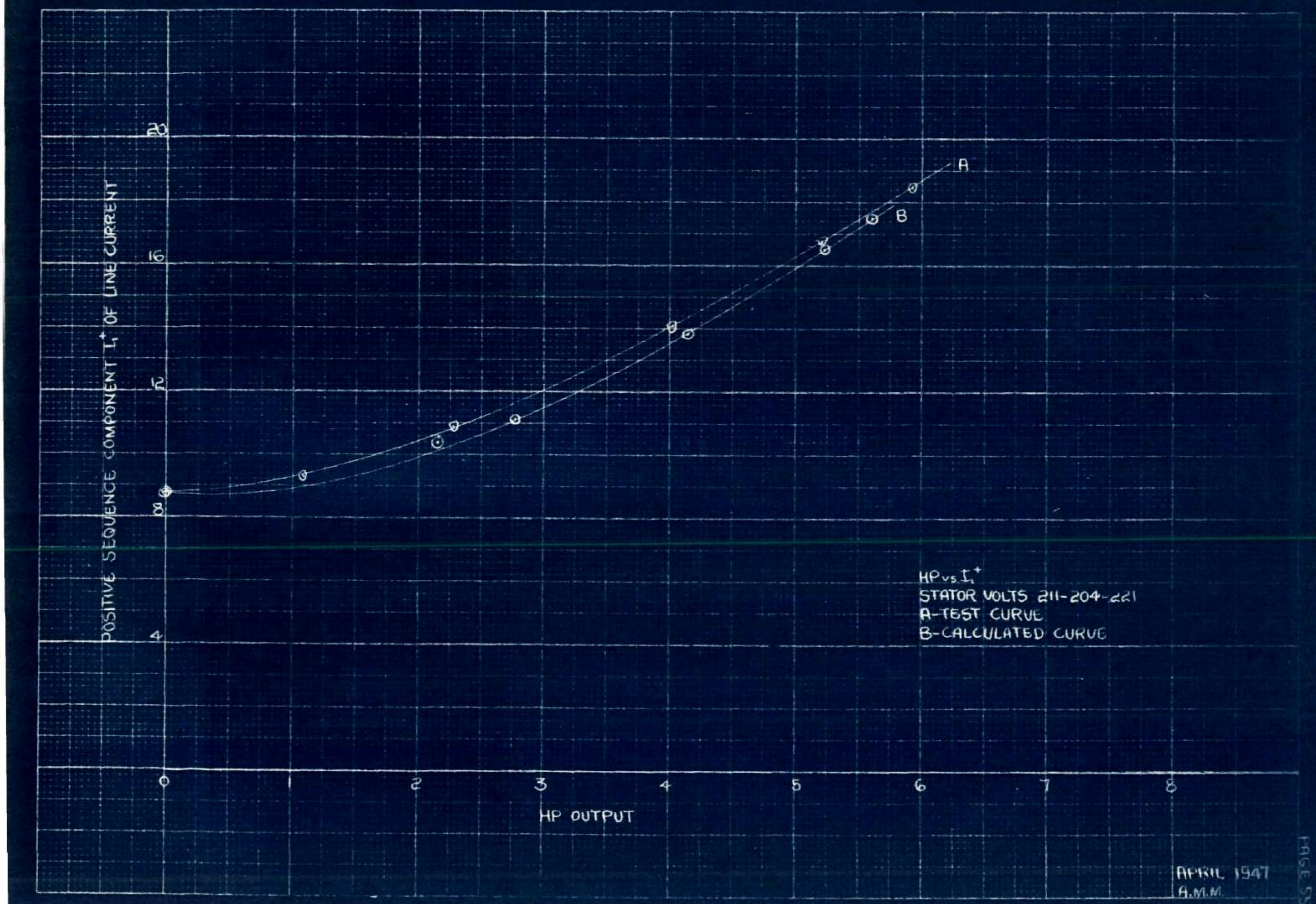

100
80
60
40
20
EFFICIENCY IN PERCENT
0
1
2
3
4
5
6
7
8
H.P OUTPUT
A
B
HP OUTPUT vs EFFICIENCY
A-BALANCED THREE PHASE
B-SINGLE PHASE
APRIL 1947
A.M.M.



SLIP IN PERCENT
10
8
6
4
2
0
1
2
3
4
5
6
7
8
HORSE POWER OUTPUT
A
B
HP vs SLIP
A-BALANCED THREE PHASE
B-SINGLE PHASE
APRIL 1947
A.M.M.



POSITIVE SEQUENCE COMPONENT I_1^+ OF LINE CURRENT
20
16
12
8
4
A
B
0
2
3
4
5
6
7
8
HP OUTPUT
HP vs I_1^+
STATOR VOLTS 213-209-224
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.


POSITIVE SEQUENCE COMPONENT I_1^+ OF LINE CURRENT
20
16
12
8
4
0
1
2
3
4
5
6
7
8
HP OUTPUT
A
B
HP vs I_1^+
STATOR VOLTS 211-204-221
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.
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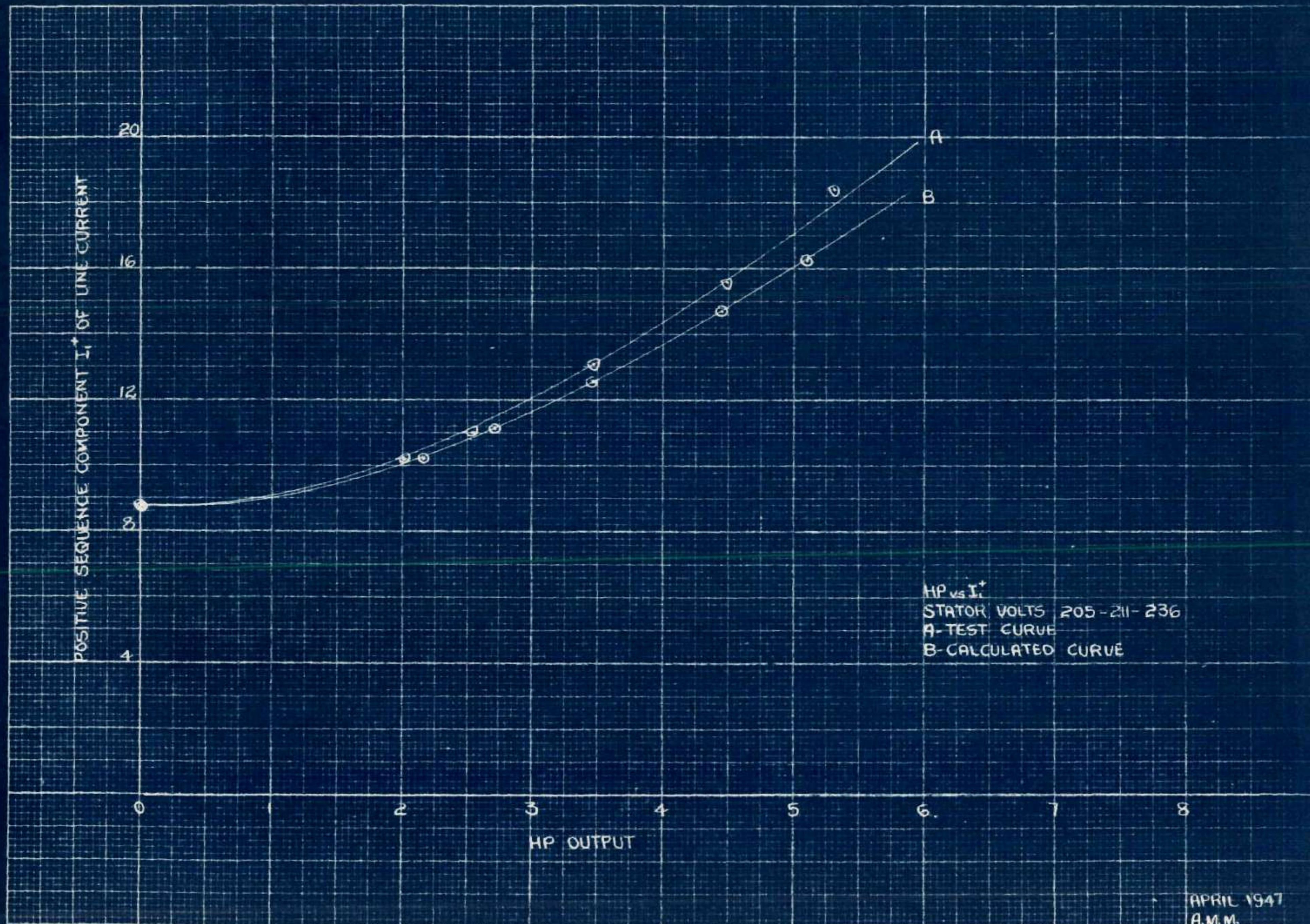
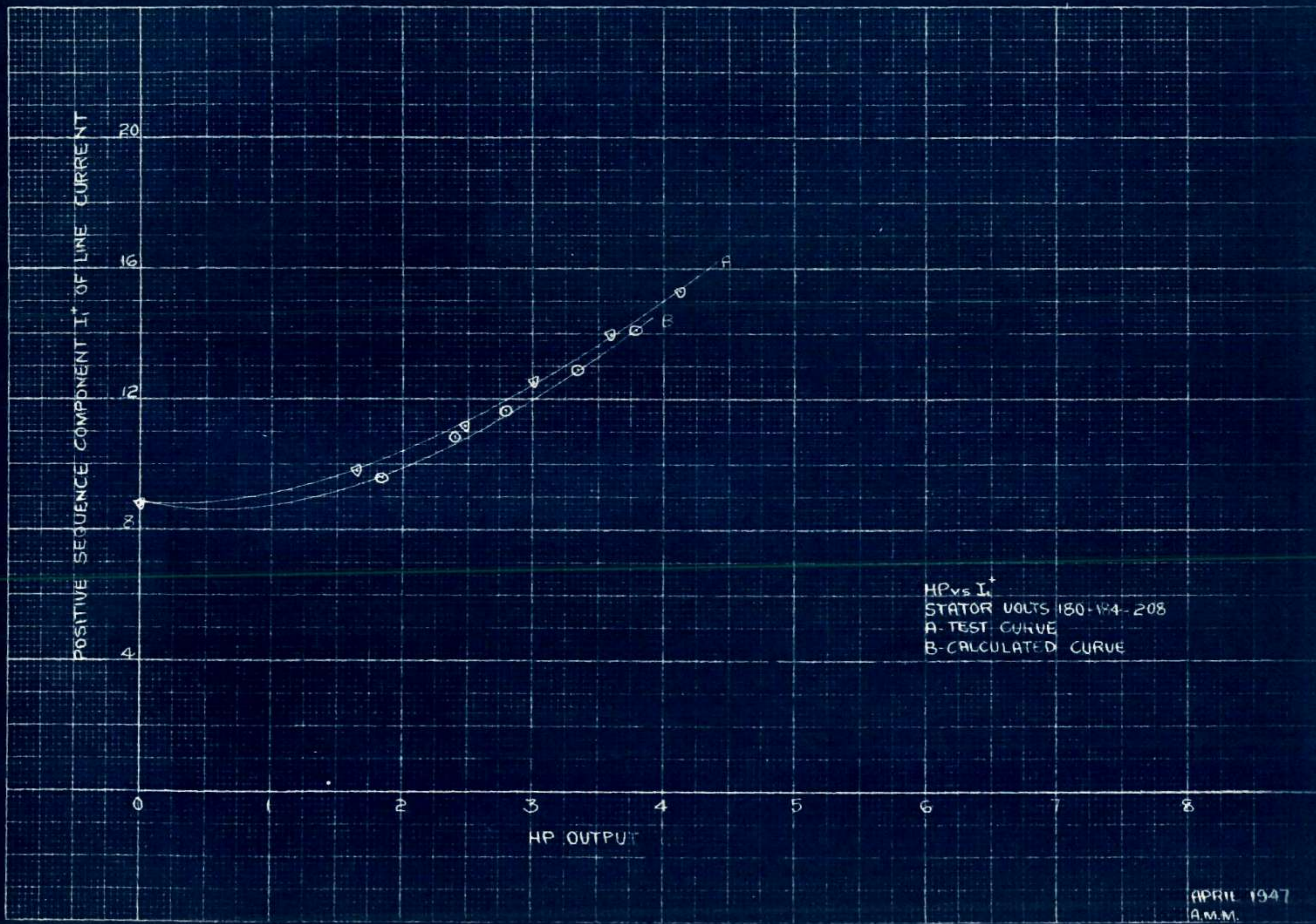
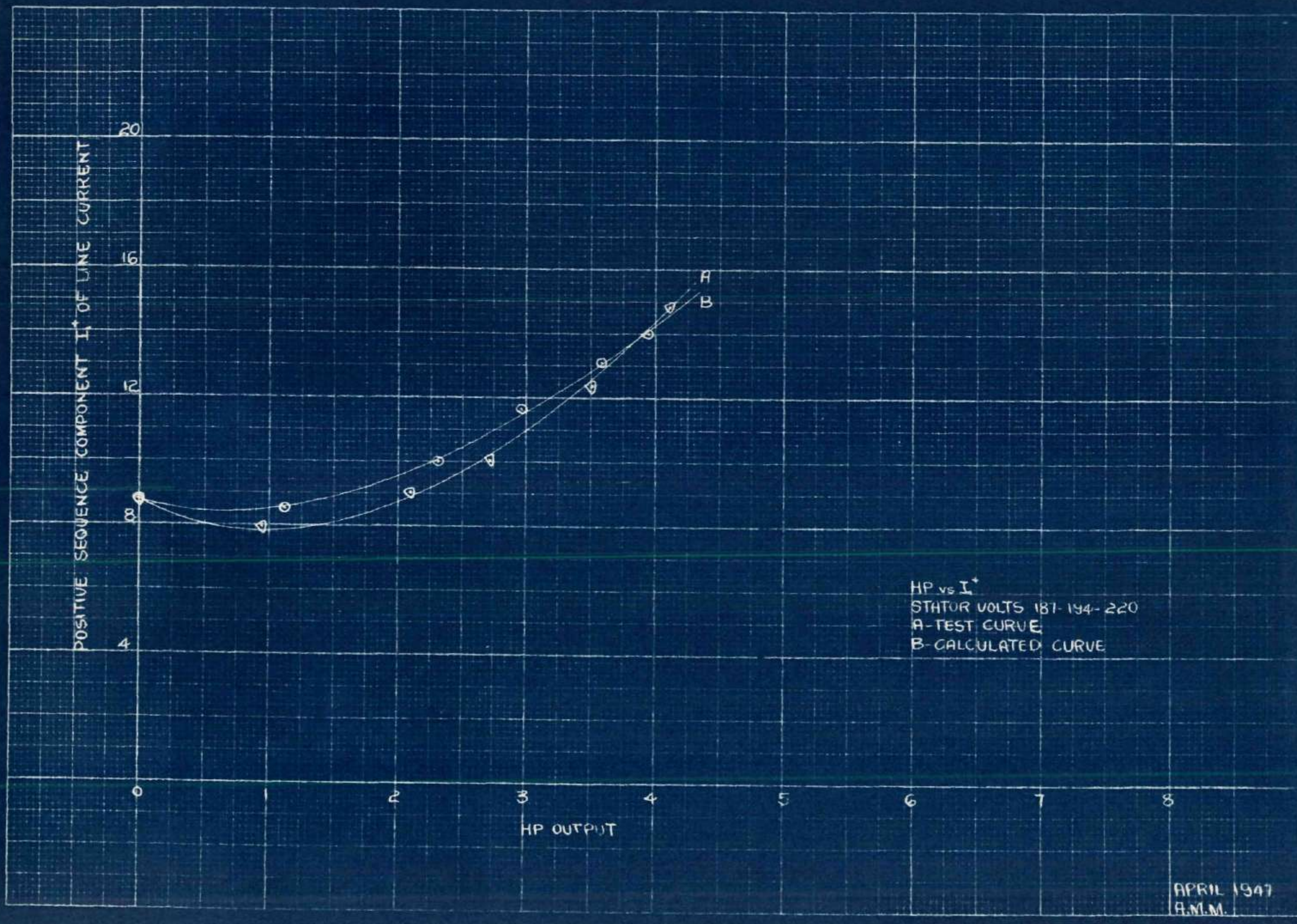


POSITIVE SEQUENCE COMPONENT I_1^+ OF LINE CURRENT
20
16
12
8
4
A
B
0
1
2
3
4
5
6.
7
8
HP OUTPUT
HP vs I_1^+
STATOR VOLTS 205 - 211 - 236
A - TEST CURVE
B - CALCULATED CURVE
APRIL 1947
A.M.M.
PAGE 56


POSITIVE SEQUENCE COMPONENT I_1^+ OF LINE CURRENT
20
16
12
8
4
0
1
2
3
4
5
6
7
8
HP OUTPUT
A
B
HP vs I_1^+
STATOR VOLTS 180-194-208
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.



POSITIVE SEQUENCE COMPONENT I_1^+ OF LINE CURRENT
20
16
12
8
4
A
B
0
1
2
3
4
5
6
7
8
HP OUTPUT
HP vs I_1^+
STATOR VOLTS 187-194-220
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.

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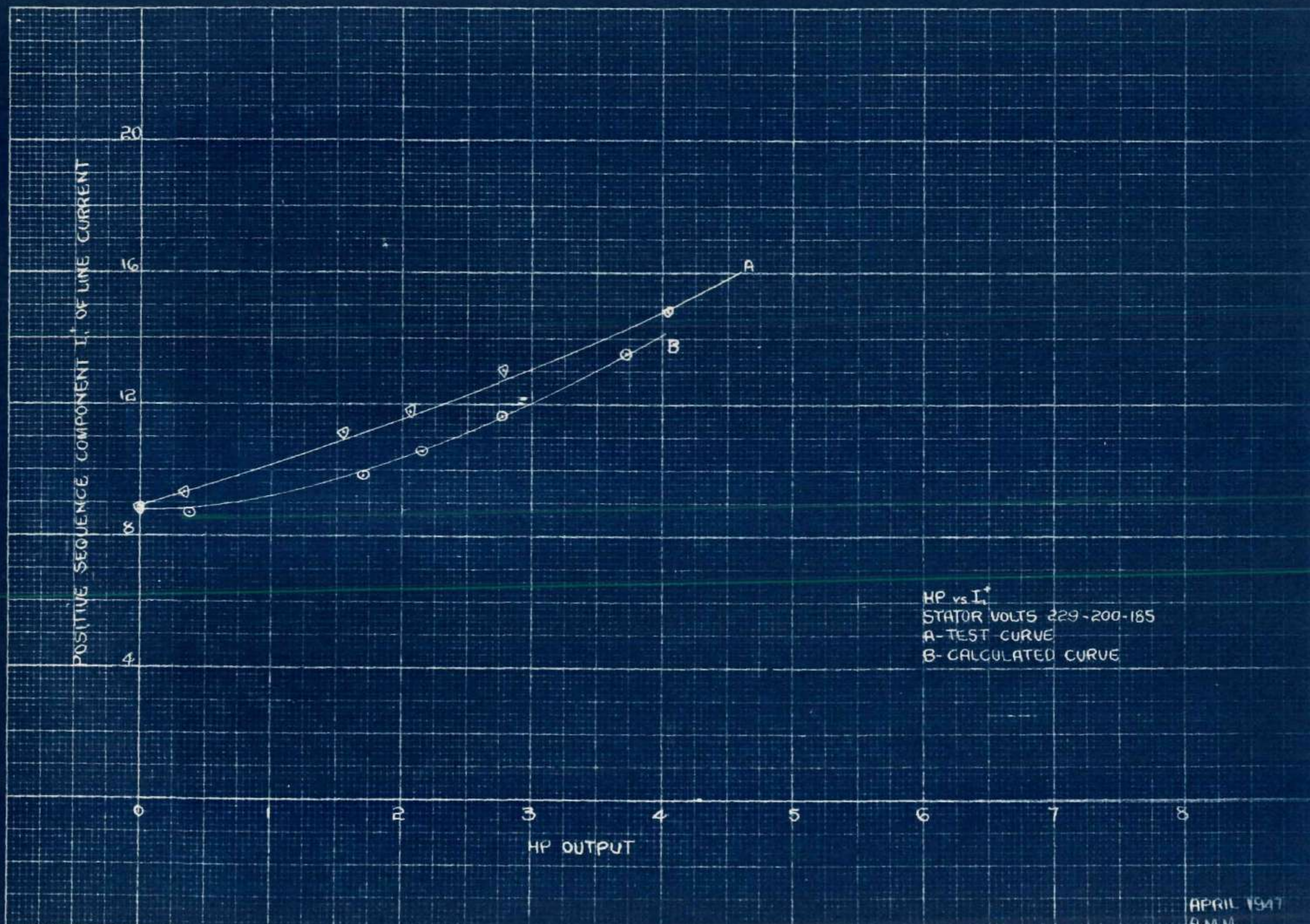
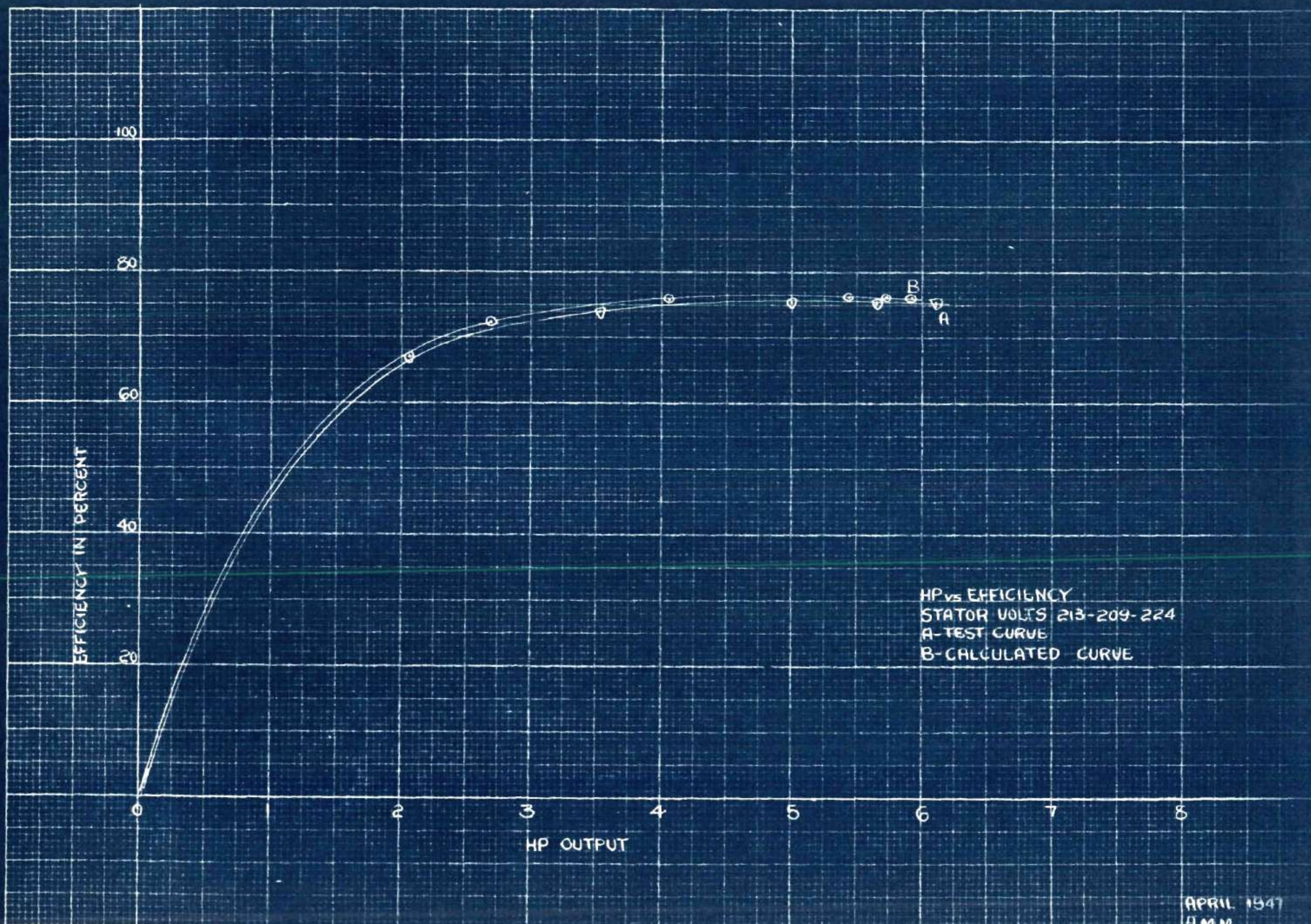
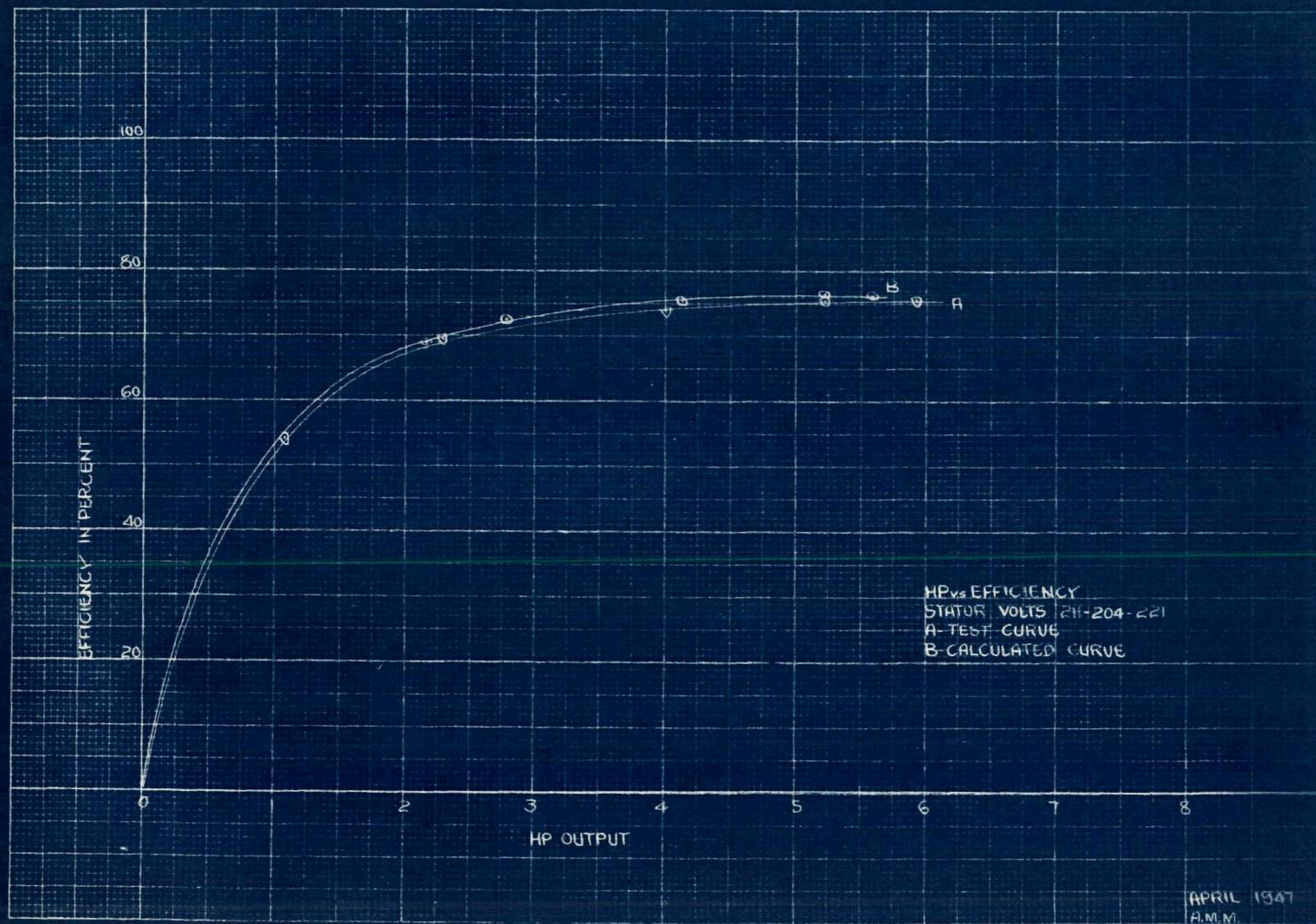
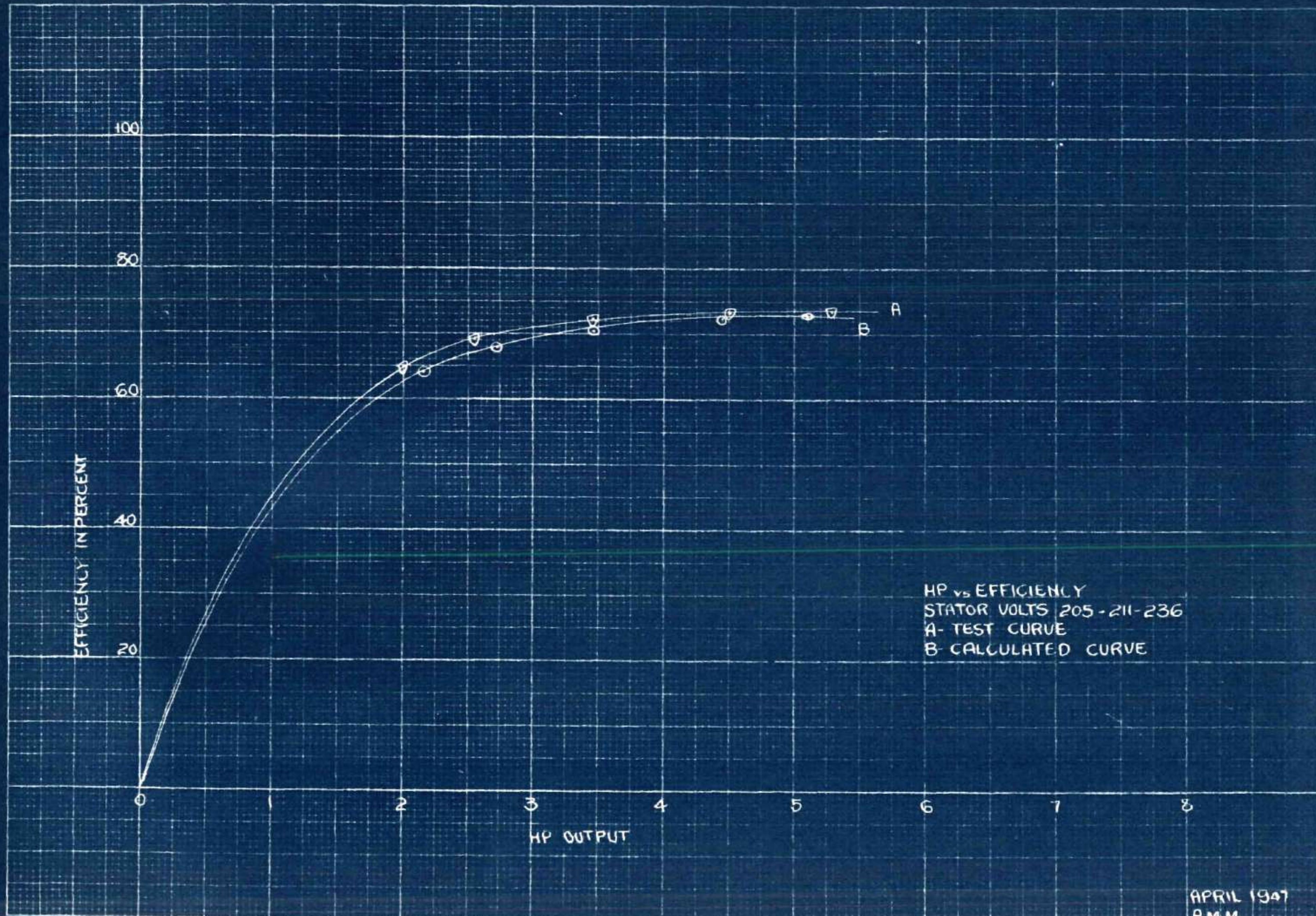

POSITIVE SEQUENCE COMPONENT I_1^+ OF LINE CURRENT
20
16
12
8
4
0
1
2
3
4
5
6
7
8
HP OUTPUT
A
B
HP vs I_1^+
STATOR VOLTS 229-200-185
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.



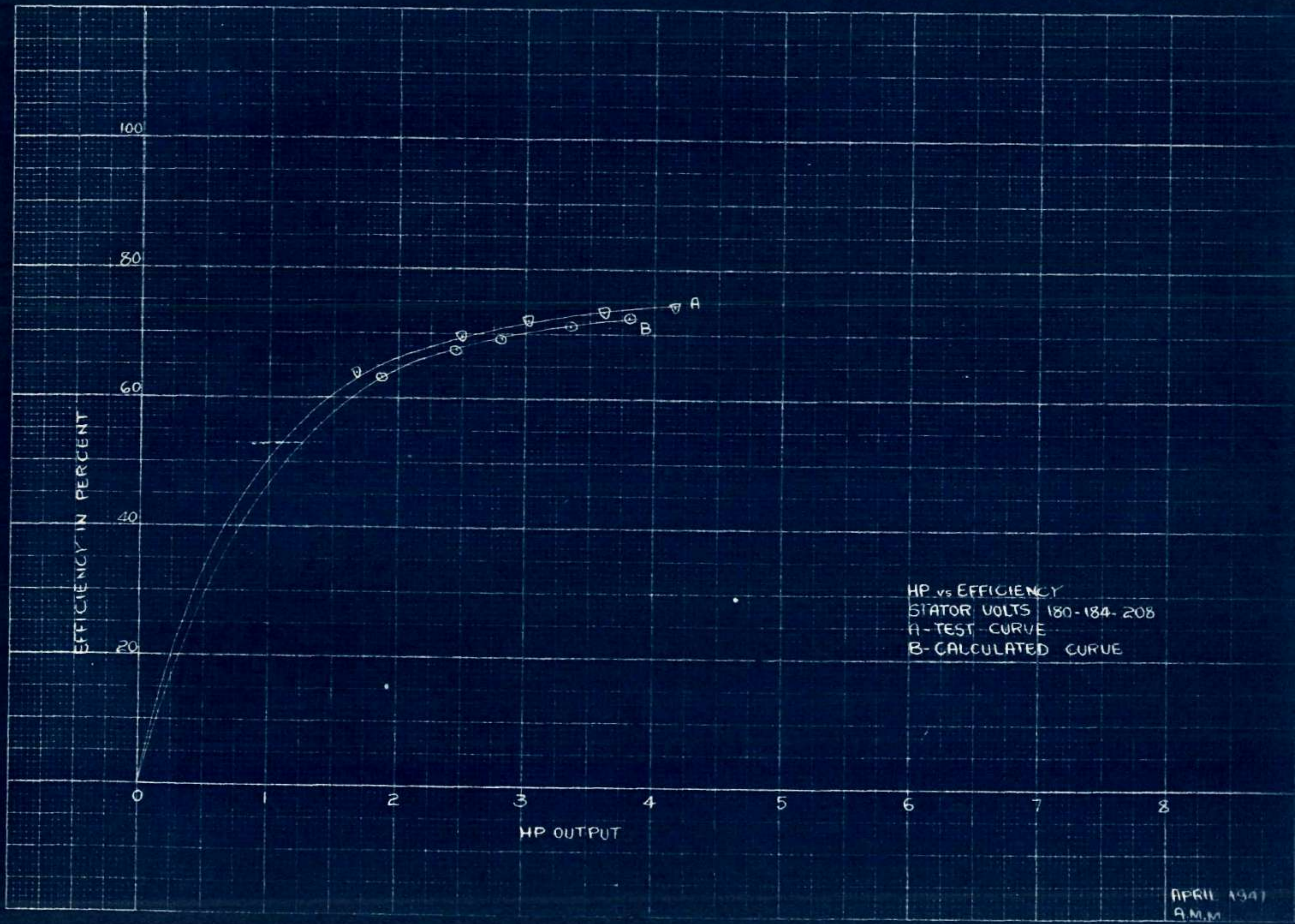
EFFICIENCY IN PERCENT
100
80
60
40
20
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs EFFICIENCY
STATOR VOLTS 213-209-224
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.
PAGE 68



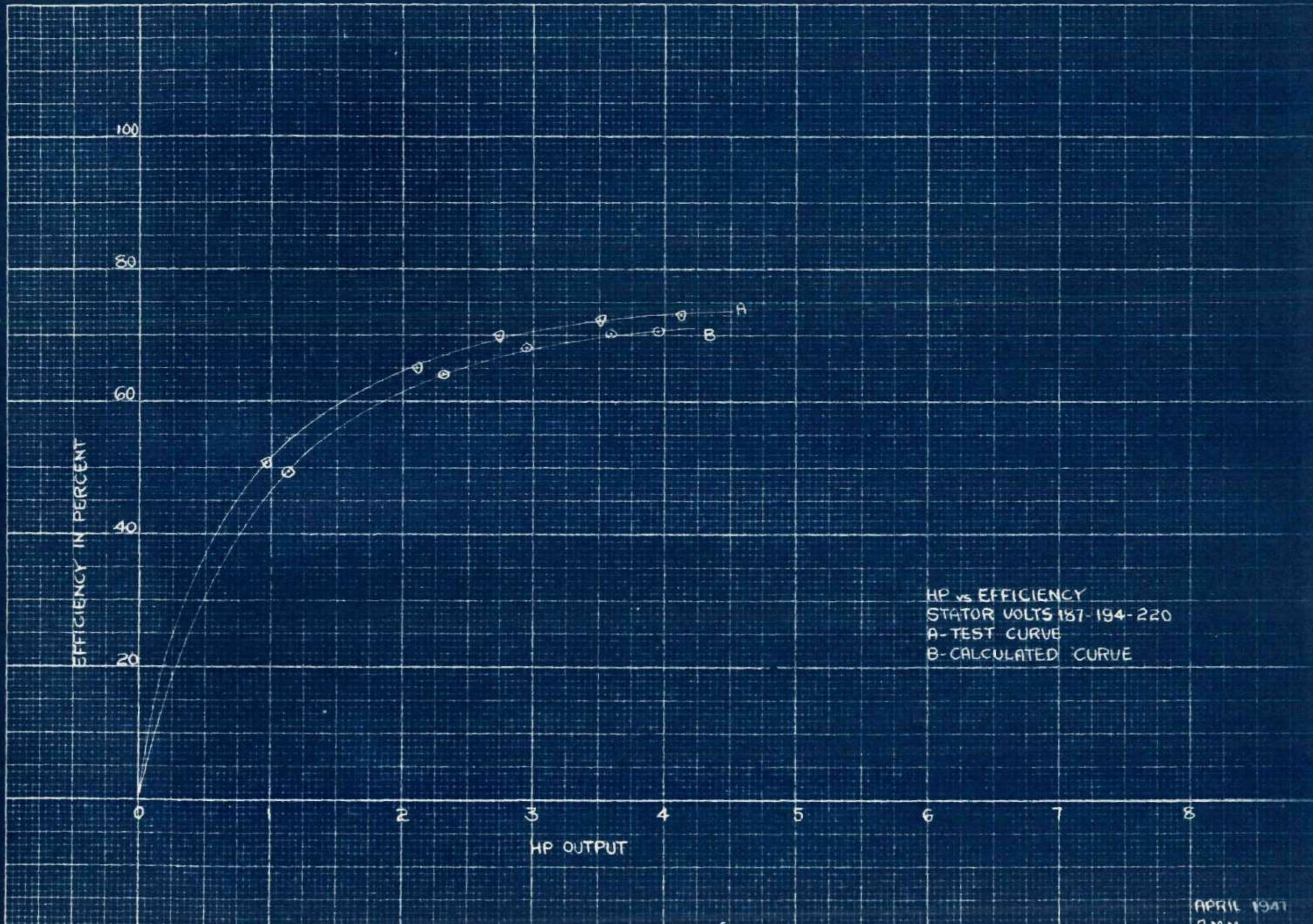
EFFICIENCY IN PERCENT
100
80
60
40
20
0
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs EFFICIENCY
STATOR VOLTS 211-204-221
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.



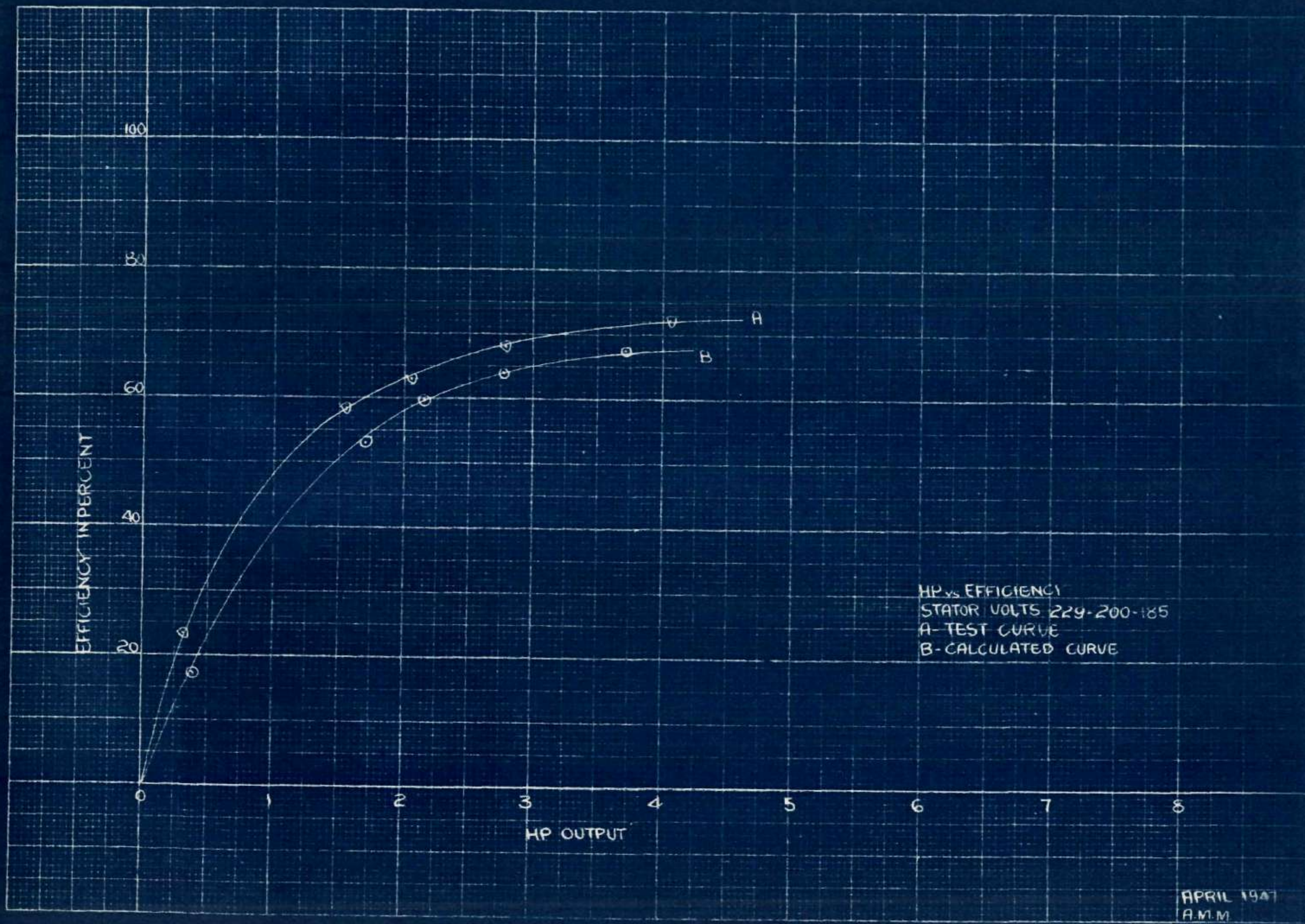
100
80
60
40
20
EFFICIENCY IN PERCENT
0
1
2
3
4
5
6
7
8
HP OUTPUT
A
B
HP vs EFFICIENCY
STATOR VOLTS 205 - 211 - 236
A- TEST CURVE
B- CALCULATED CURVE
APRIL 1947
A.M.M.



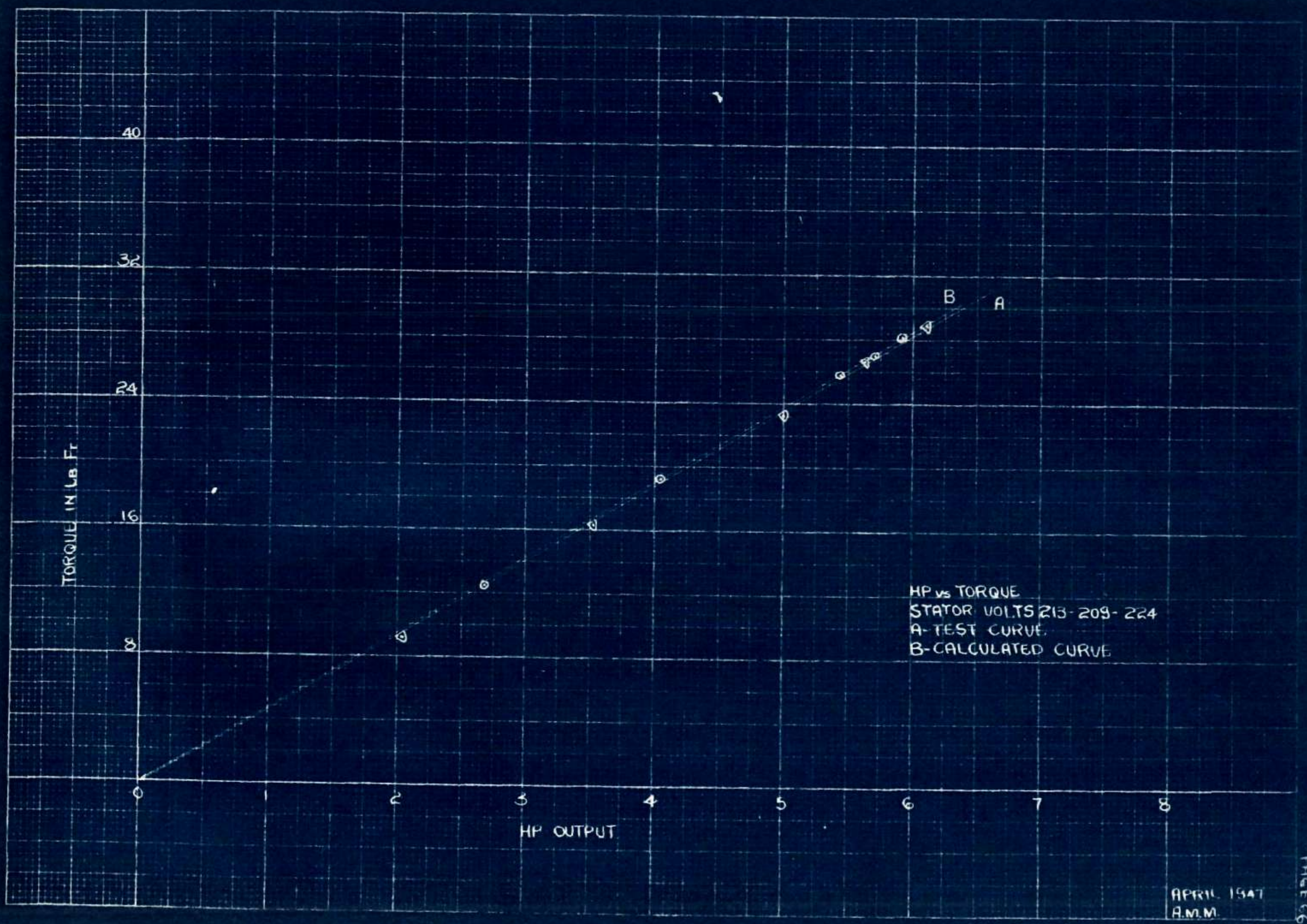
HP vs EFFICIENCY
STATOR VOLTS 180-184-208
A-TEST CURVE
B-CALCULATED CURVE
EFFICIENCY IN PERCENT
100
80
60
40
20
0
1
2
3
4
5
6
7
8
HP OUTPUT
A
B
APRIL 1941
A.M.M

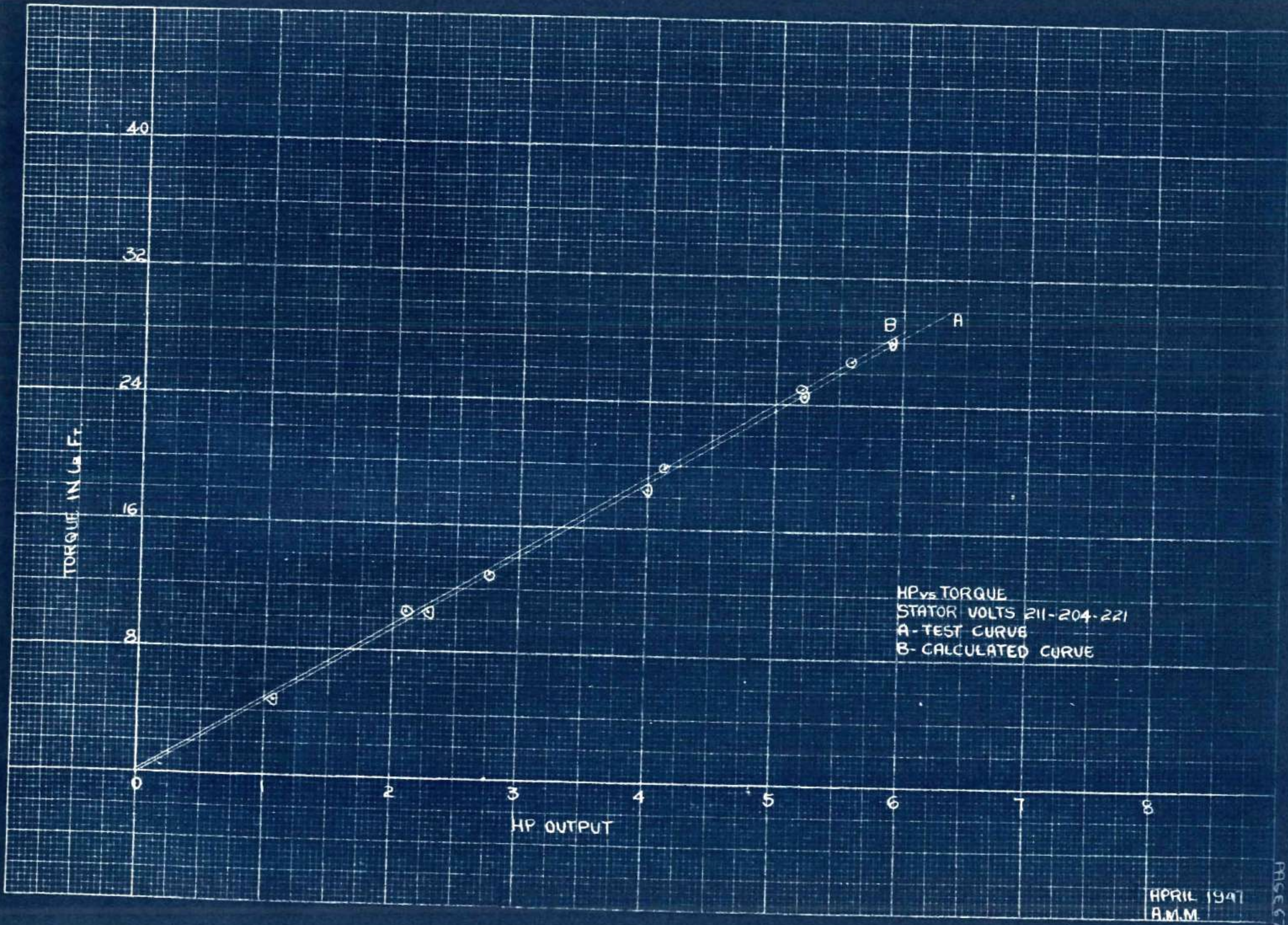


100
80
60
40
20
EFFICIENCY IN PERCENT
0
1
2
3
4
5
6
7
8
HP OUTPUT
A
B
HP vs EFFICIENCY
STATOR VOLTS 187-194-220
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.
PAGE 69

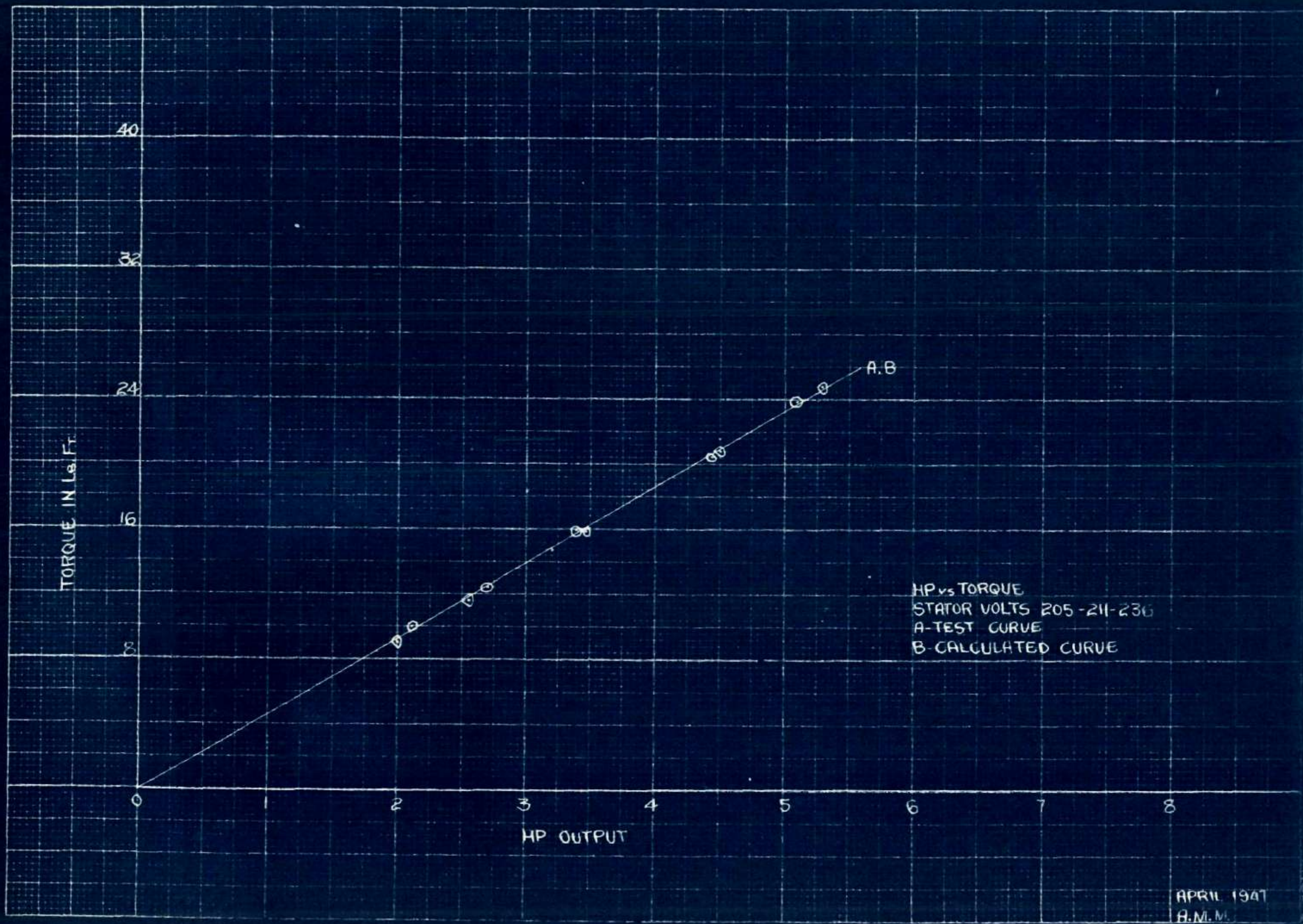


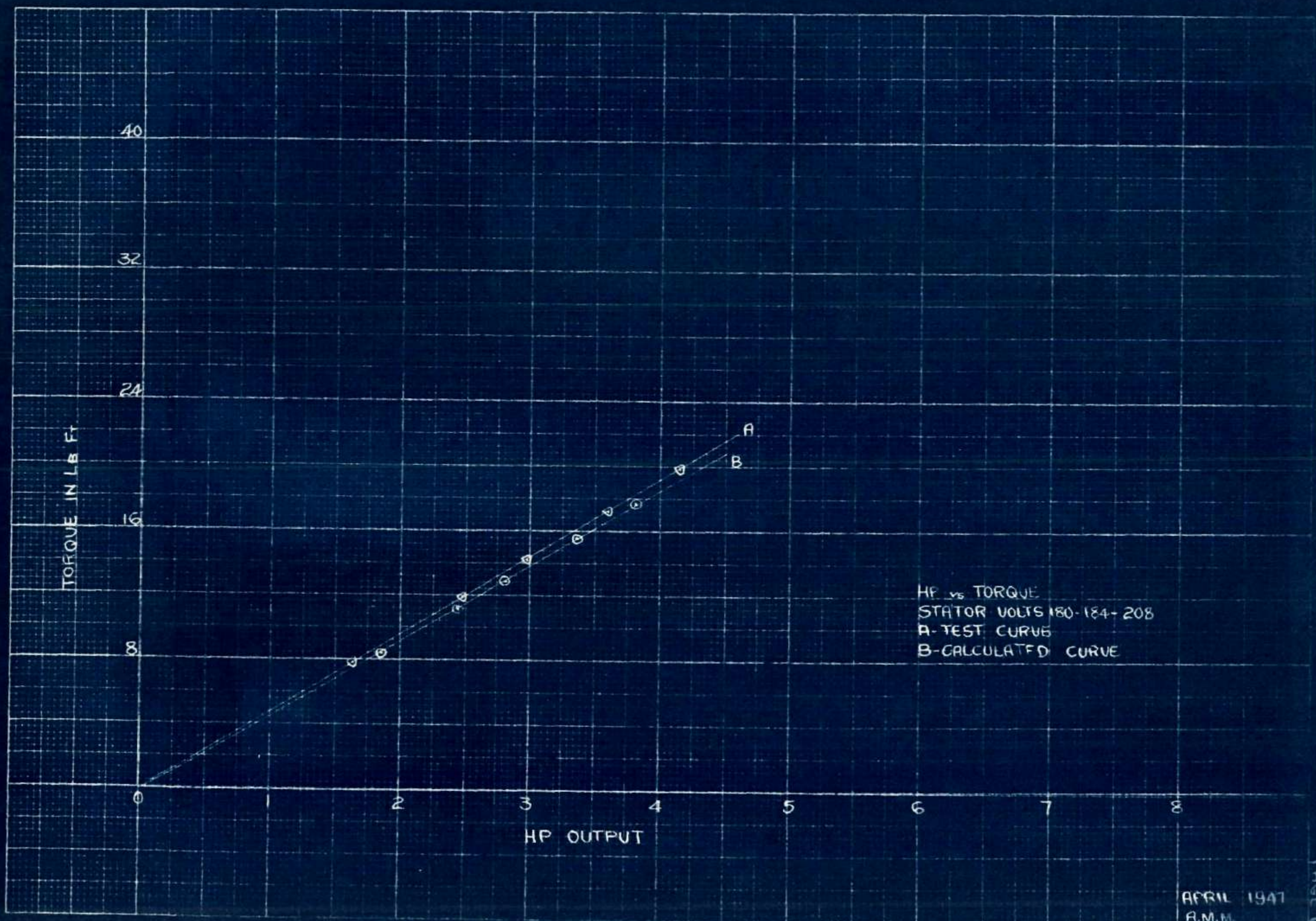
100
80
60
40
20
EFFICIENCY IN PERCENT
0
1
2
3
4
5
6
7
8
HP OUTPUT
A
B
HP vs EFFICIENCY
STATOR VOLTS 229-200-185
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M

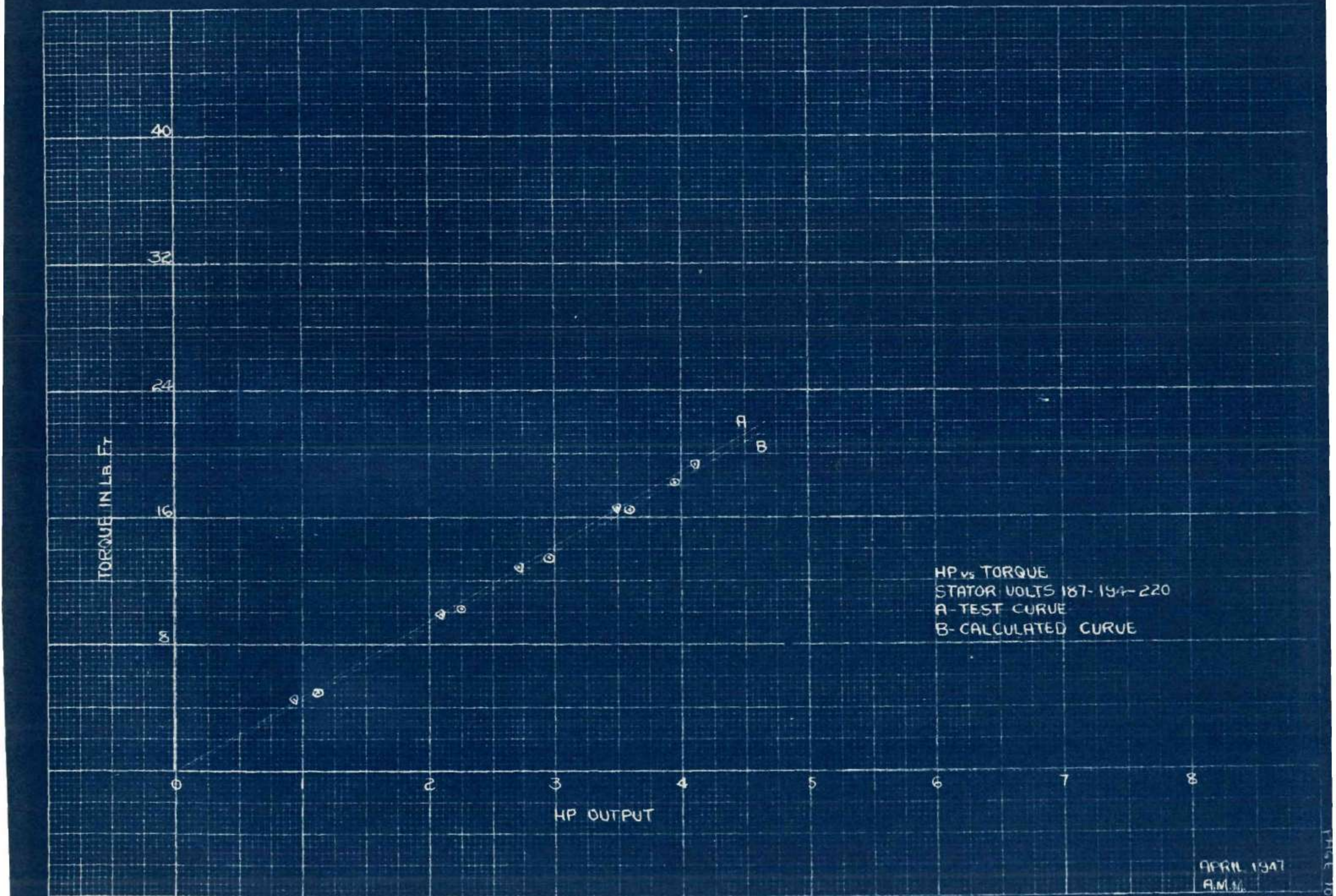

TORQUE IN LB. FT.
40
32
24
16
8
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs TORQUE
STATOR VOLTS 213-209-224
A-TEST CURVE.
B-CALCULATED CURVE
APRIL 1947
A.M.M.

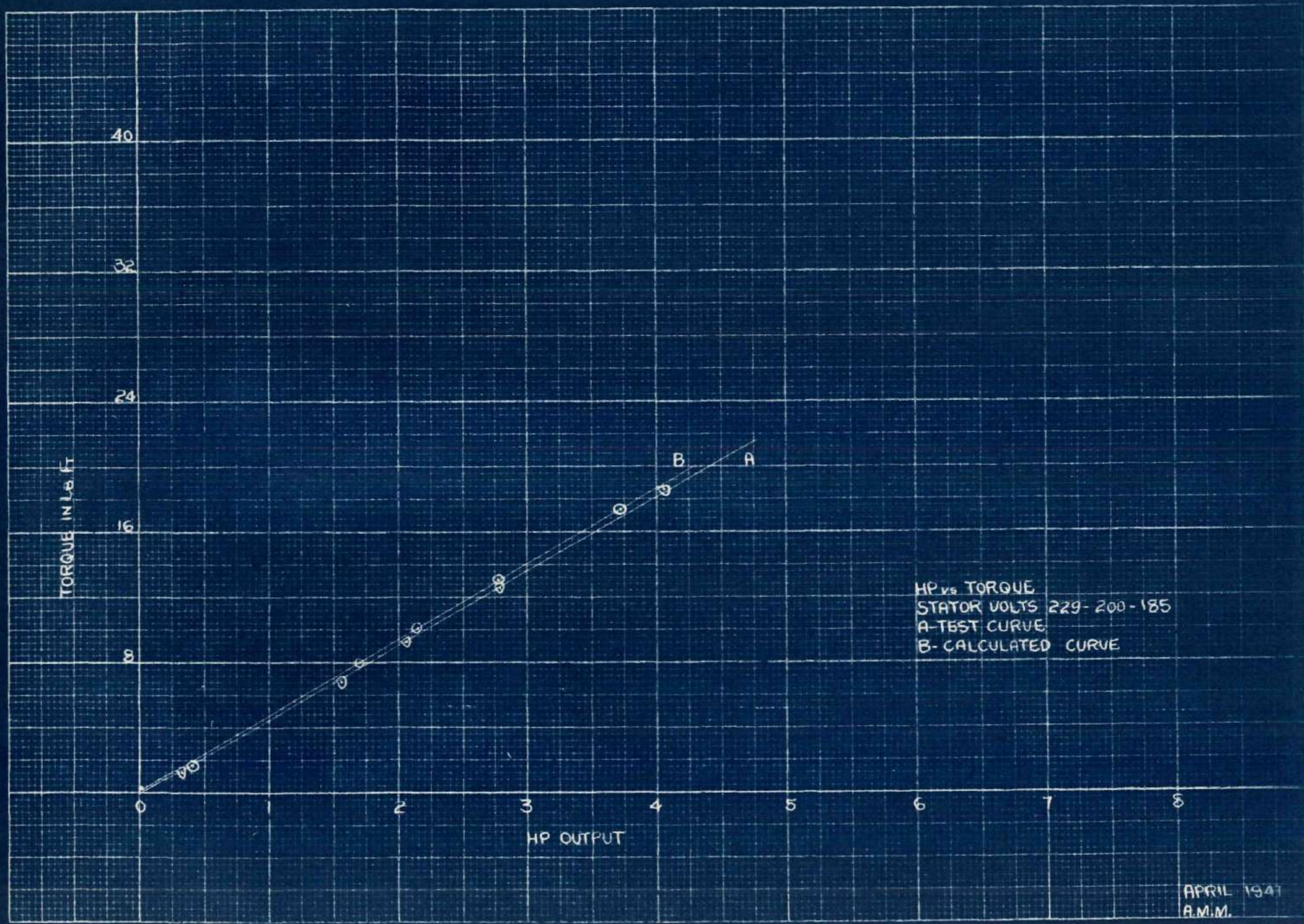


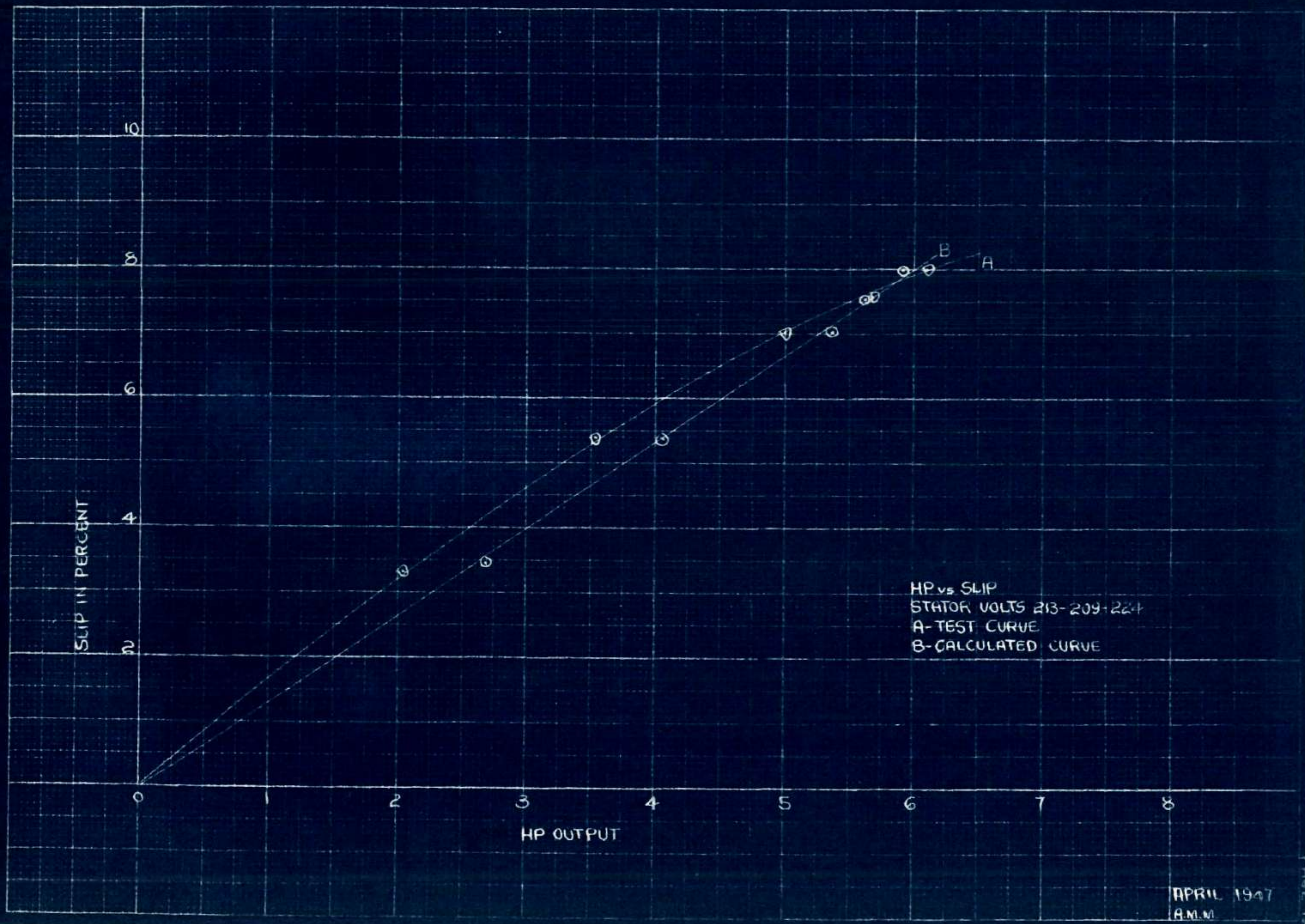
TORQUE IN Lb. Ft.
40
32
24
16
8
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs TORQUE
STATOR VOLTS 211-204-221
A - TEST CURVE
B - CALCULATED CURVE
APRIL 1947
A.M.M.
PAGE 67


TORQUE IN LB. FT.
40
32
24
16
8
0
1
2
3
4
5
6
7
8
HP OUTPUT
A.B
HP vs TORQUE
STATOR VOLTS 205-211-236
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.

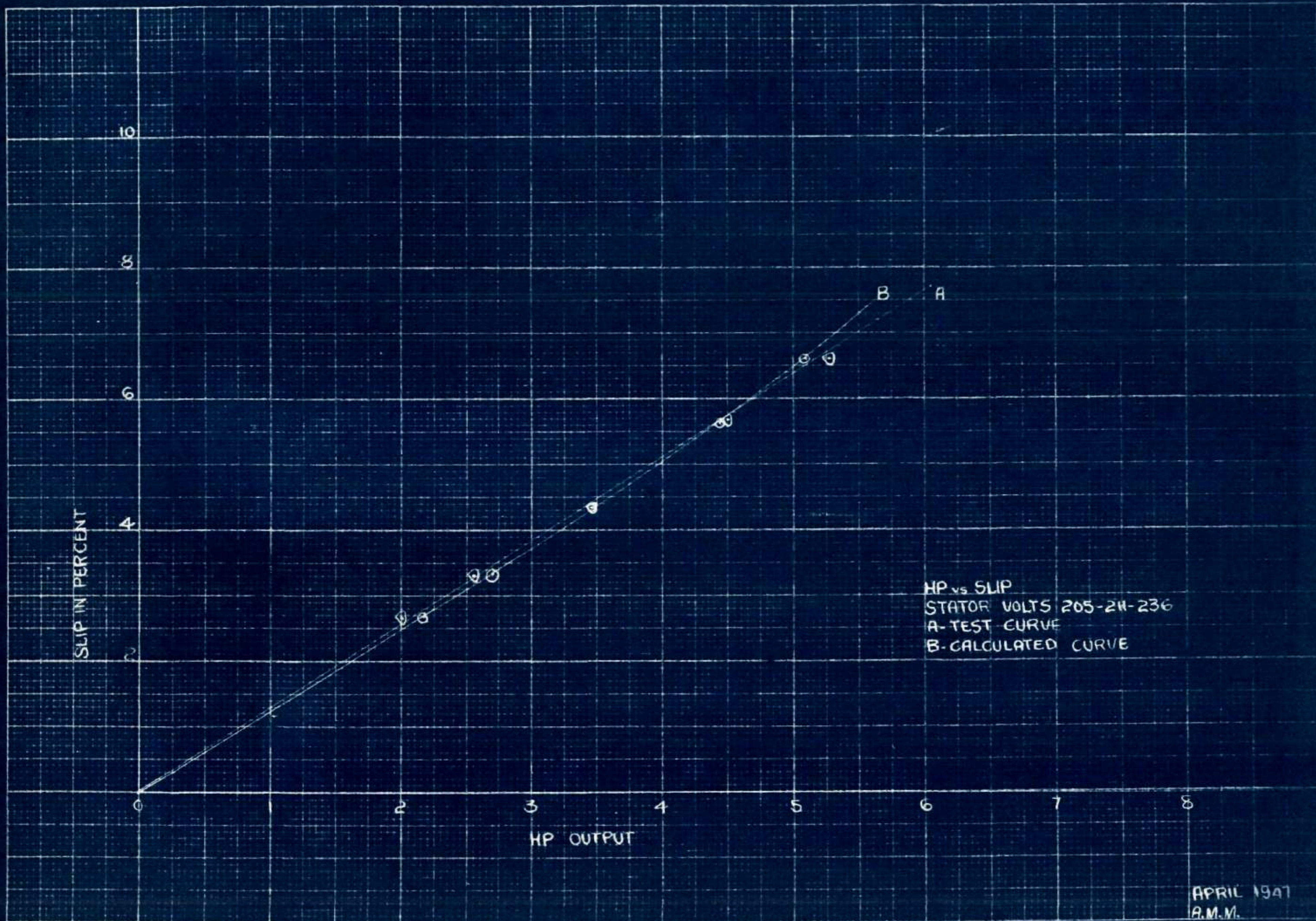

40
32
24
16
8
0
1
2
3
4
5
6
7
8
TORQUE IN LB FT
HP OUTPUT
A
B
HP vs TORQUE
STATOR VOLTS 180-184-208
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.

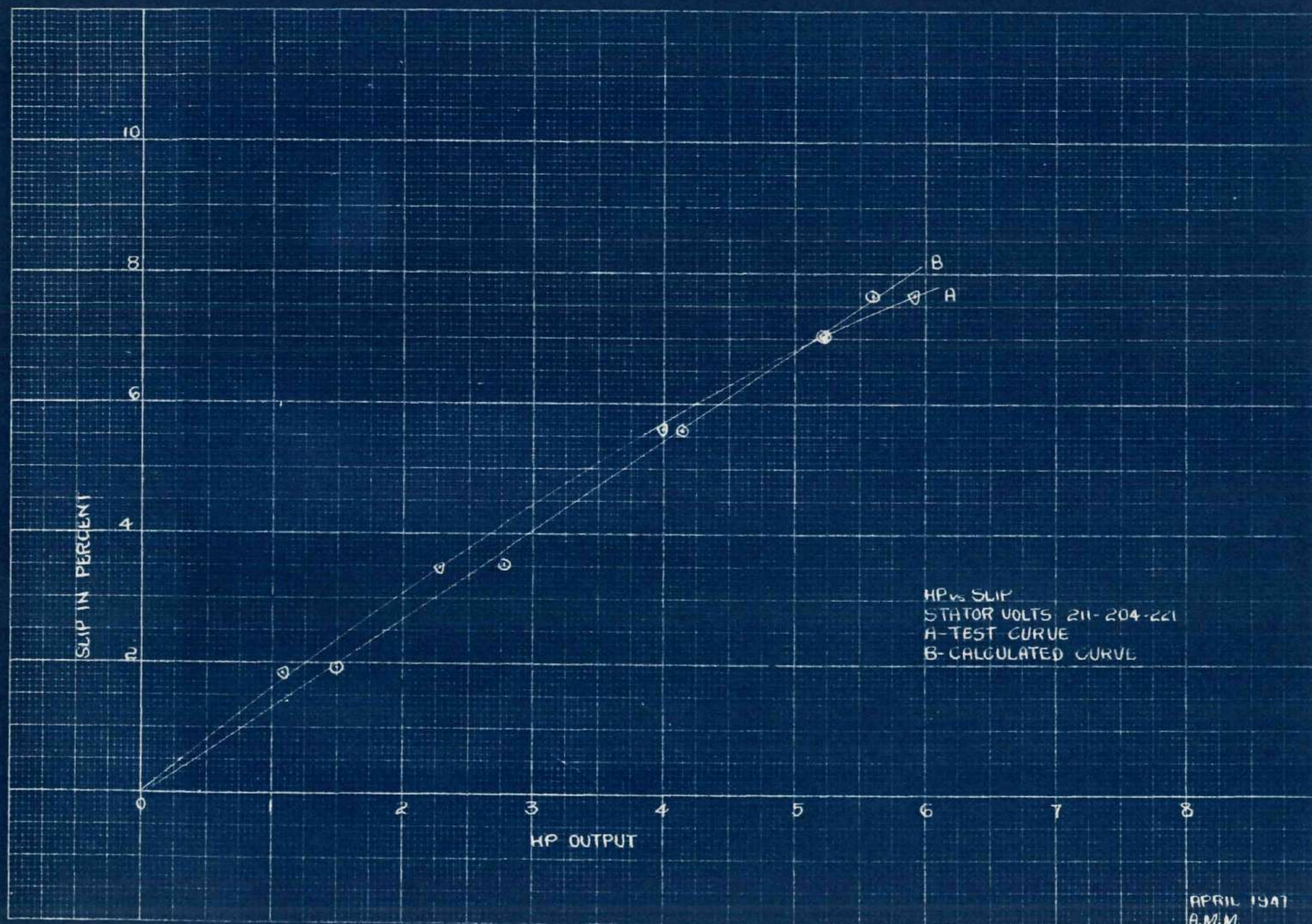

TORQUE IN LB. FT.
40
32
24
16
8
0
2
3
4
5
6
7
8
HP OUTPUT
A
B
HP vs TORQUE
STATOR VOLTS 187-194-220
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.W.
PAGE 70

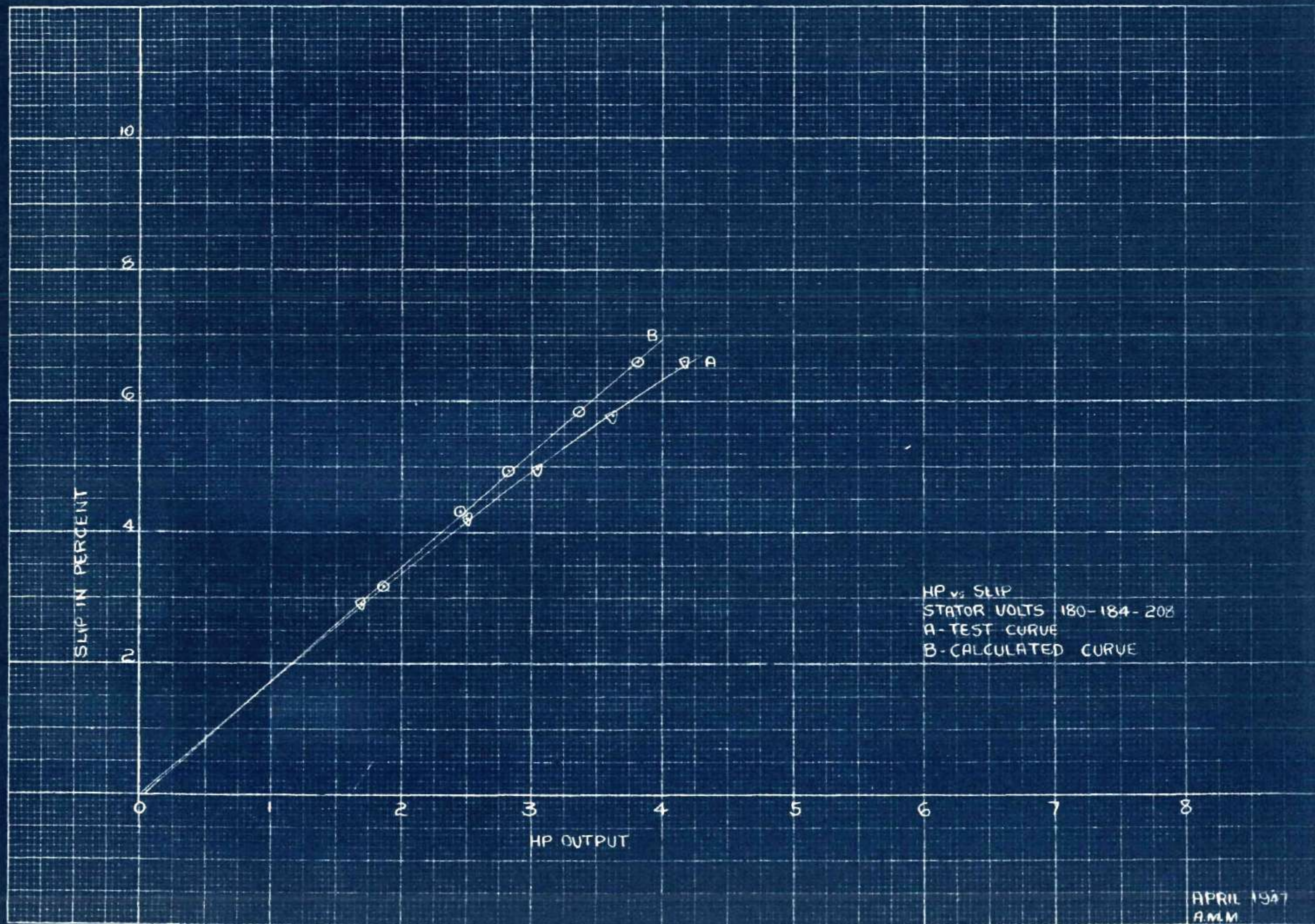

TORQUE IN Lb Ft
40
32
24
16
8
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs TORQUE
STATOR VOLTS 229-200-185
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.



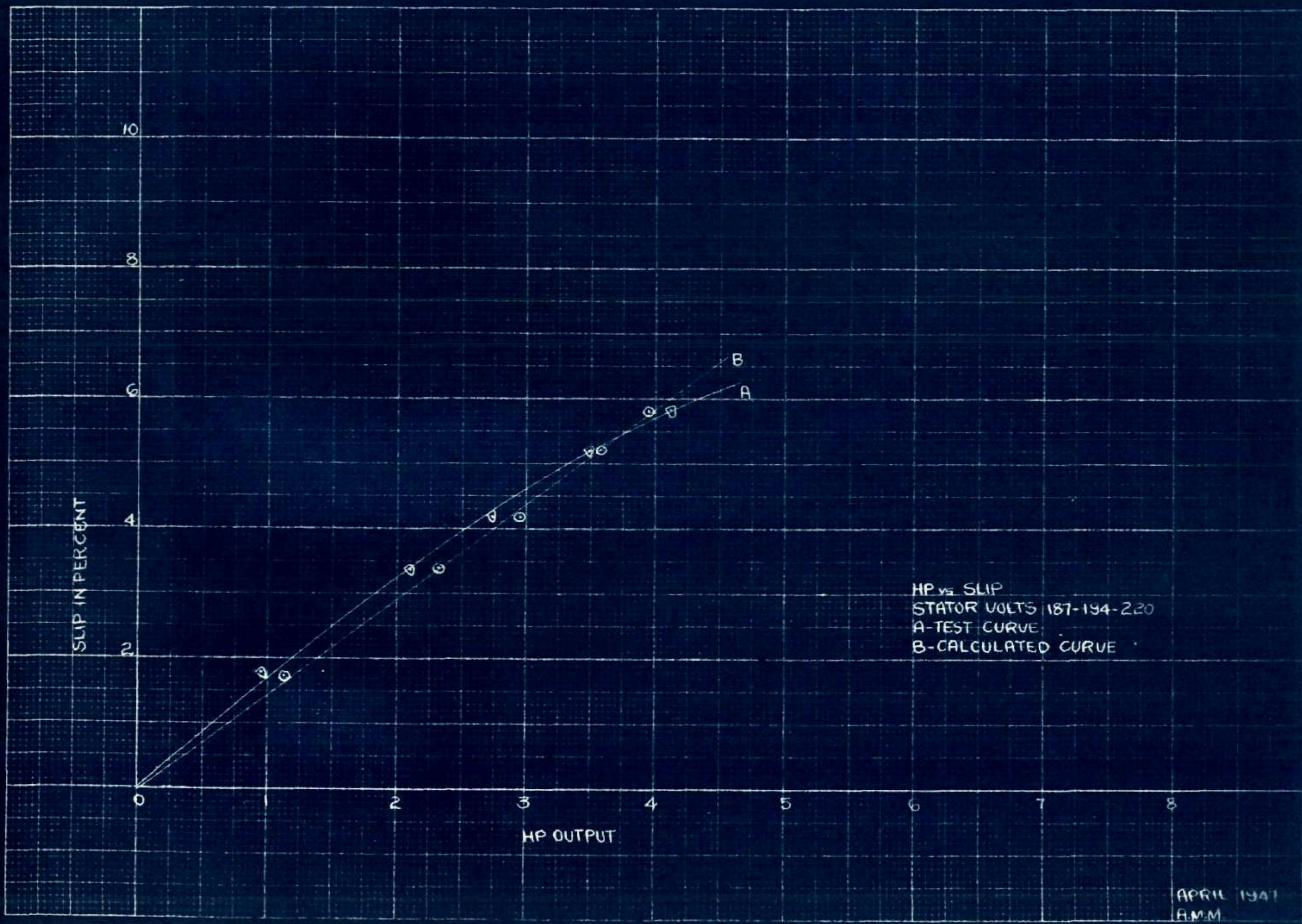
10
8
6
4
2
SLIP IN PERCENT
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs SLIP
STATOR VOLTS 213-209-224
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.

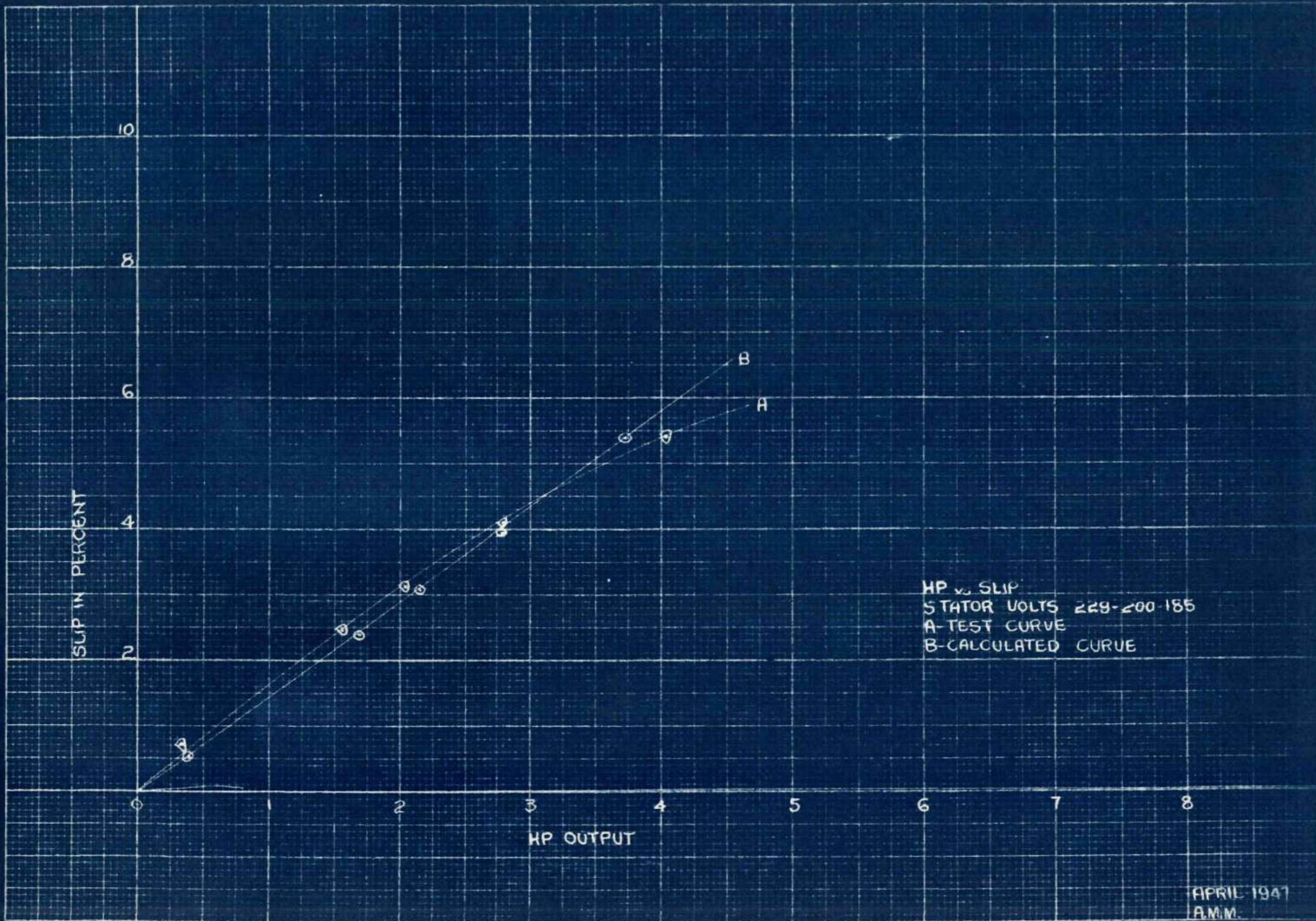

10
8
6
4
2
0
1
2
3
4
5
6
7
8
SLIP IN PERCENT
HP OUTPUT
B
A
HP vs SLIP
STATOR VOLTS 205-211-236
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.

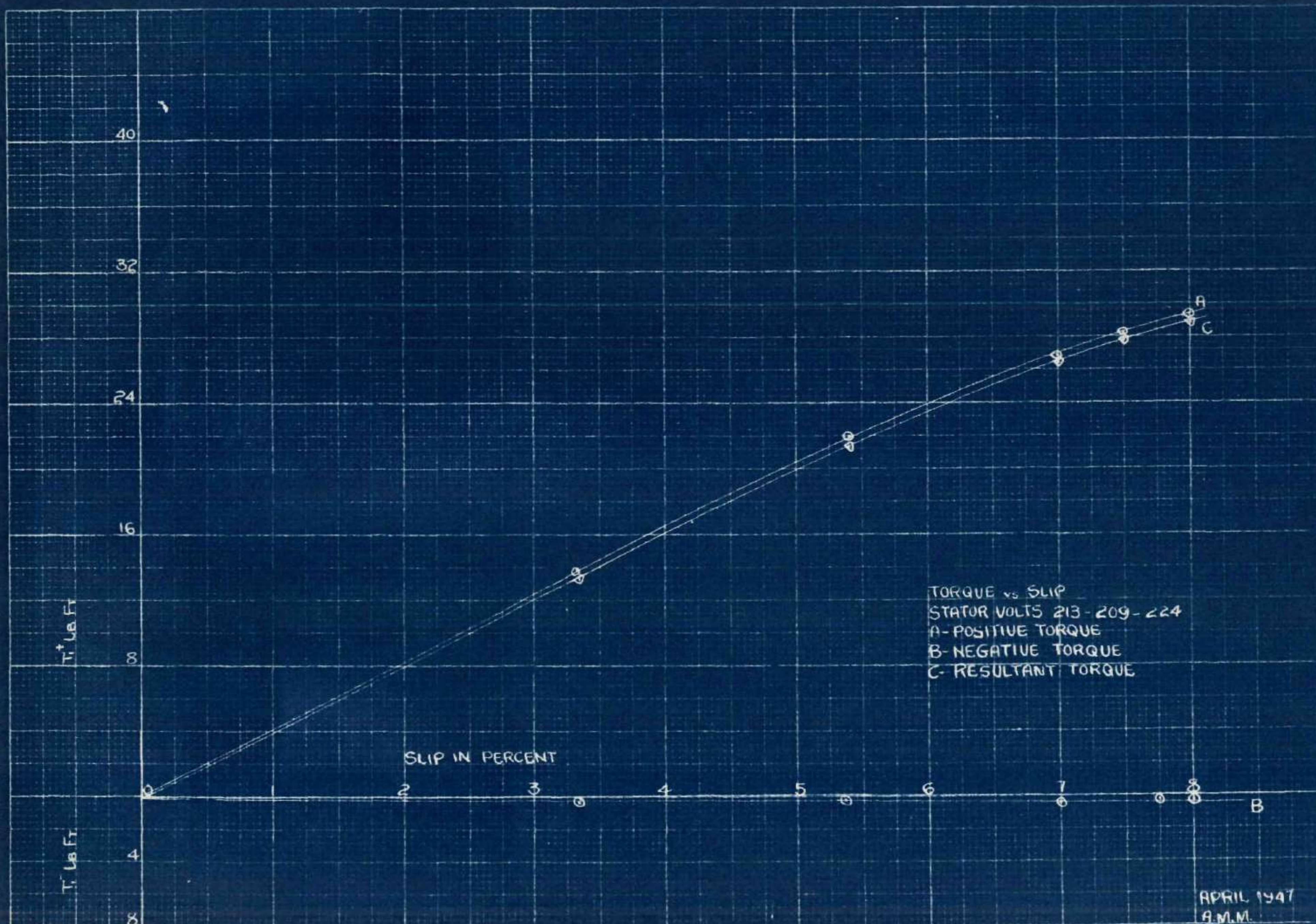

SLIP IN PERCENT
10
8
6
4
2
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs SLIP
STATOR VOLTS 211-204-221
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.



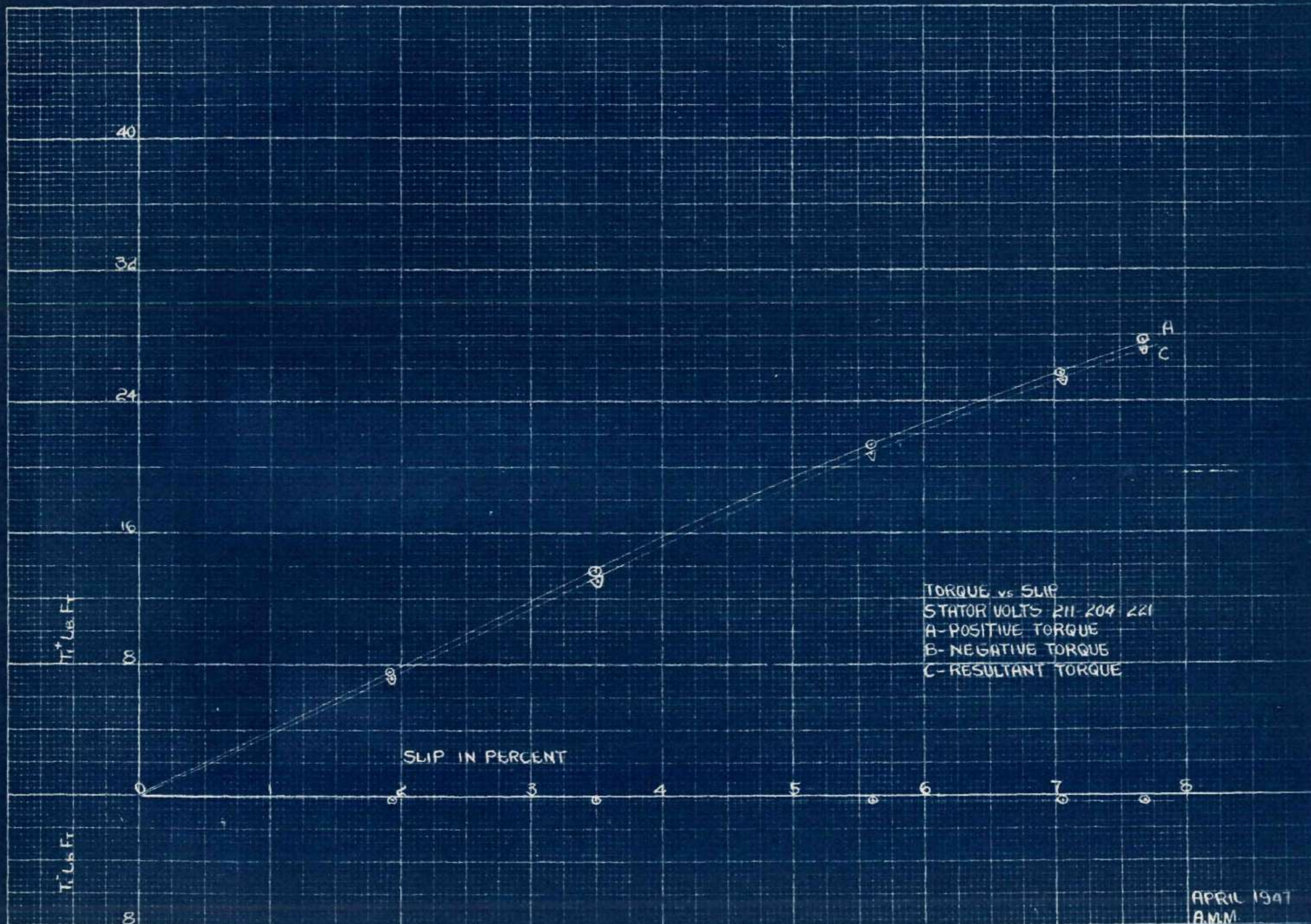
SLIP IN PERCENT
10
8
6
4
2
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs SLIP
STATOR VOLTS 180-184-208
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M


10
8
6
4
2
SLIP IN PERCENT
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs SLIP
STATOR VOLTS 187-194-220
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1947
A.M.M.

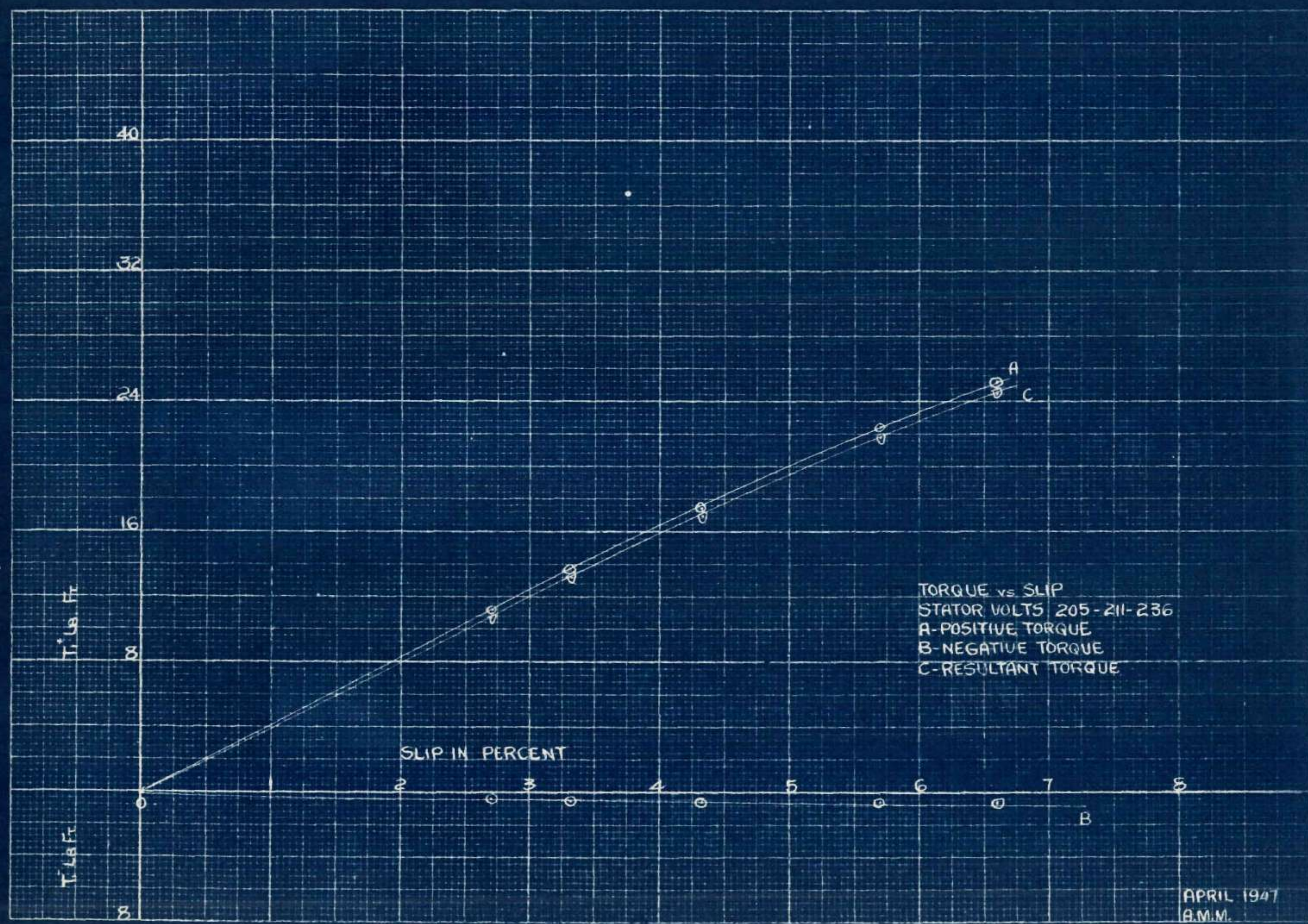

10
8
6
4
2
SLIP IN PERCENT
0
1
2
3
4
5
6
7
8
HP OUTPUT
B
A
HP vs SLIP
STATOR VOLTS 229-200-185
A-TEST CURVE
B-CALCULATED CURVE
APRIL 1941
A.M.M.

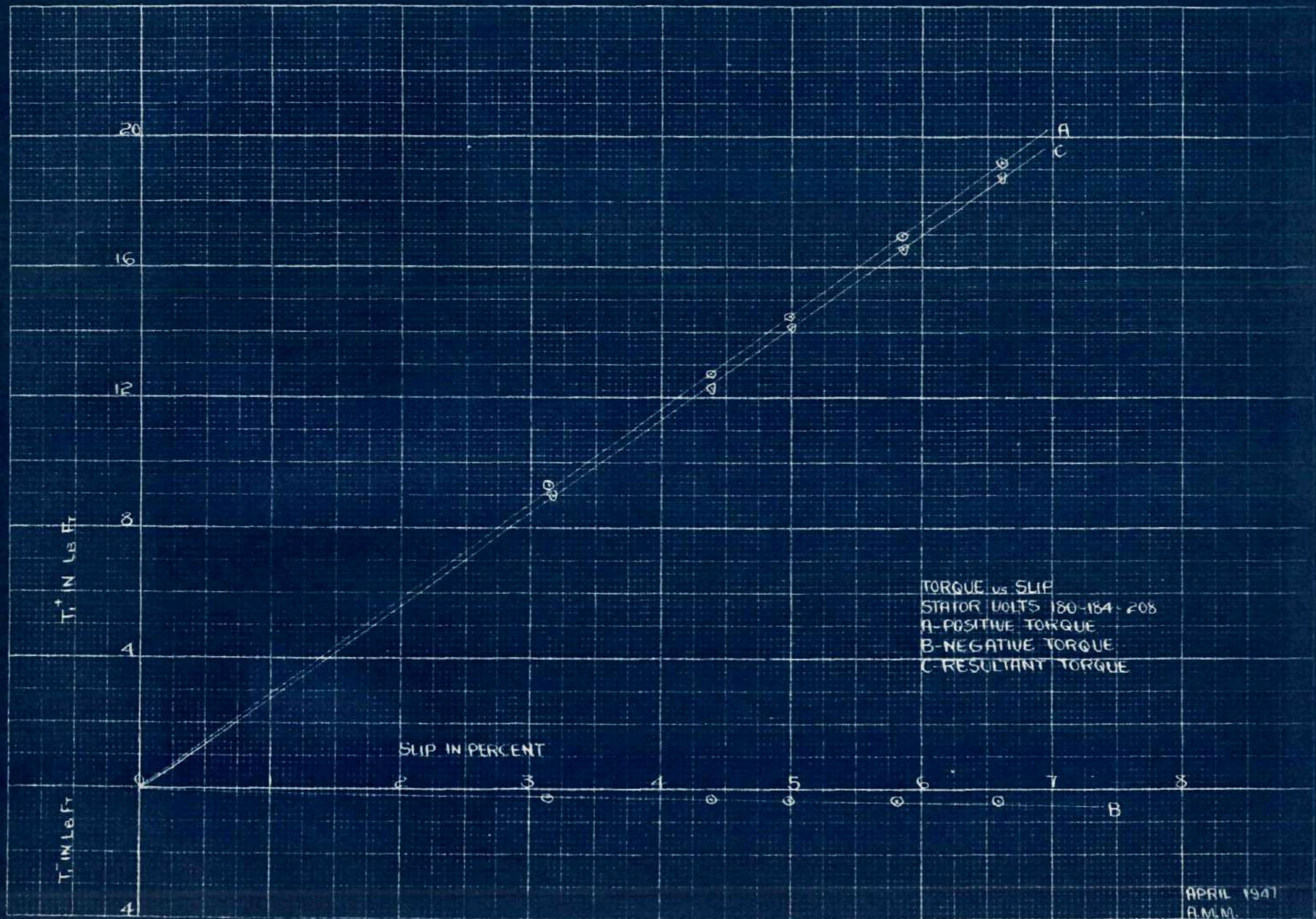


40
32
24
16
8
4
8
T^+ LB FT.
T^- LB FT.
0
1
2
3
4
5
6
7
8
SLIP IN PERCENT
A
B
C
TORQUE vs SLIP
STATOR VOLTS 213 - 209 - 224
A- POSITIVE TORQUE
B- NEGATIVE TORQUE
C- RESULTANT TORQUE
APRIL 1947
A.M.M.

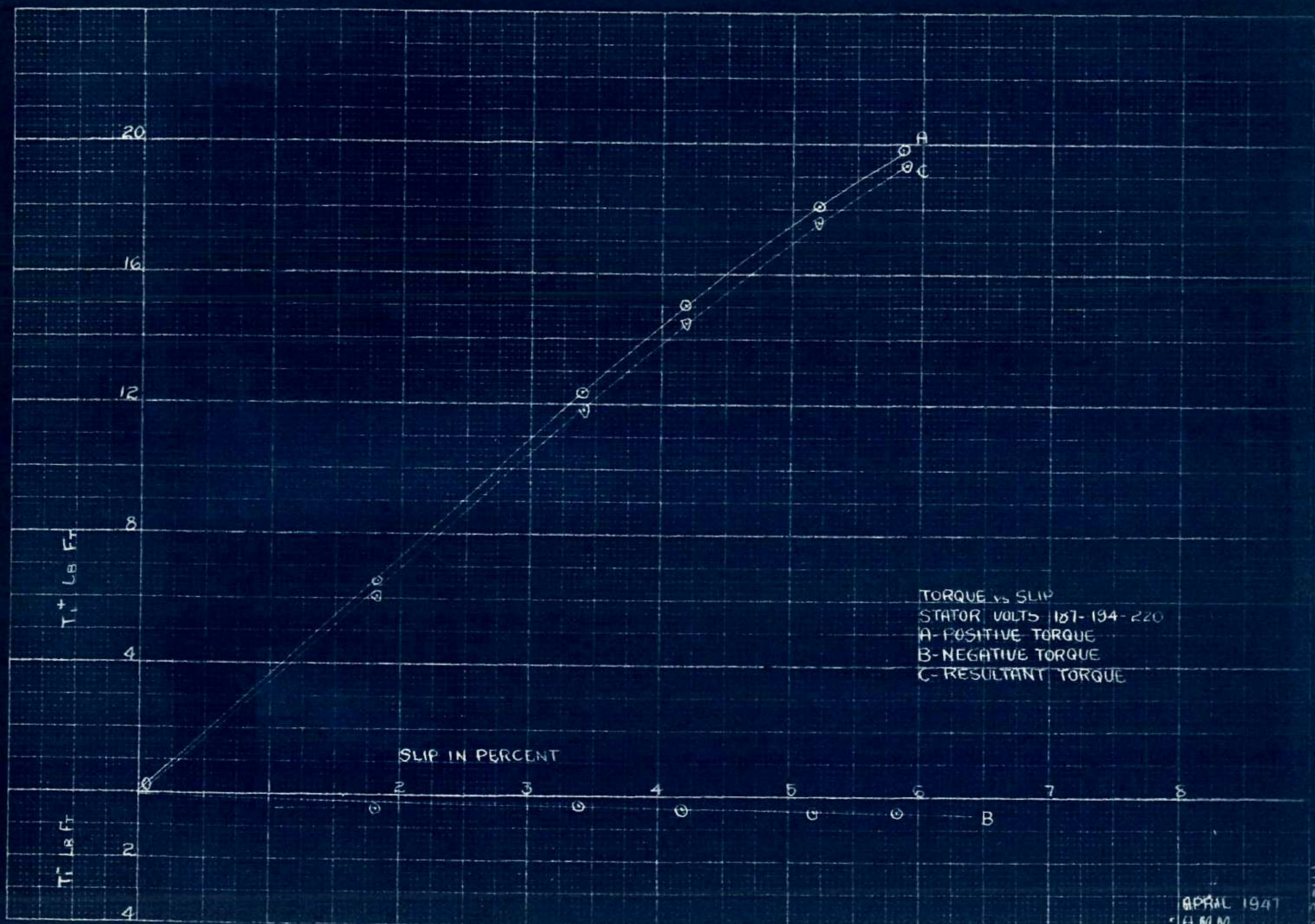


TORQUE vs SLIP
STATOR VOLTS 211 204 221
A-POSITIVE TORQUE
B-NEGATIVE TORQUE
C-RESULTANT TORQUE
SLIP IN PERCENT
A
C
40
32
24
16
8
0
1
2
3
4
5
6
7
8
8
T_1^+ Lb Ft
T_1^- Lb Ft
APRIL 1947
A.M.M.

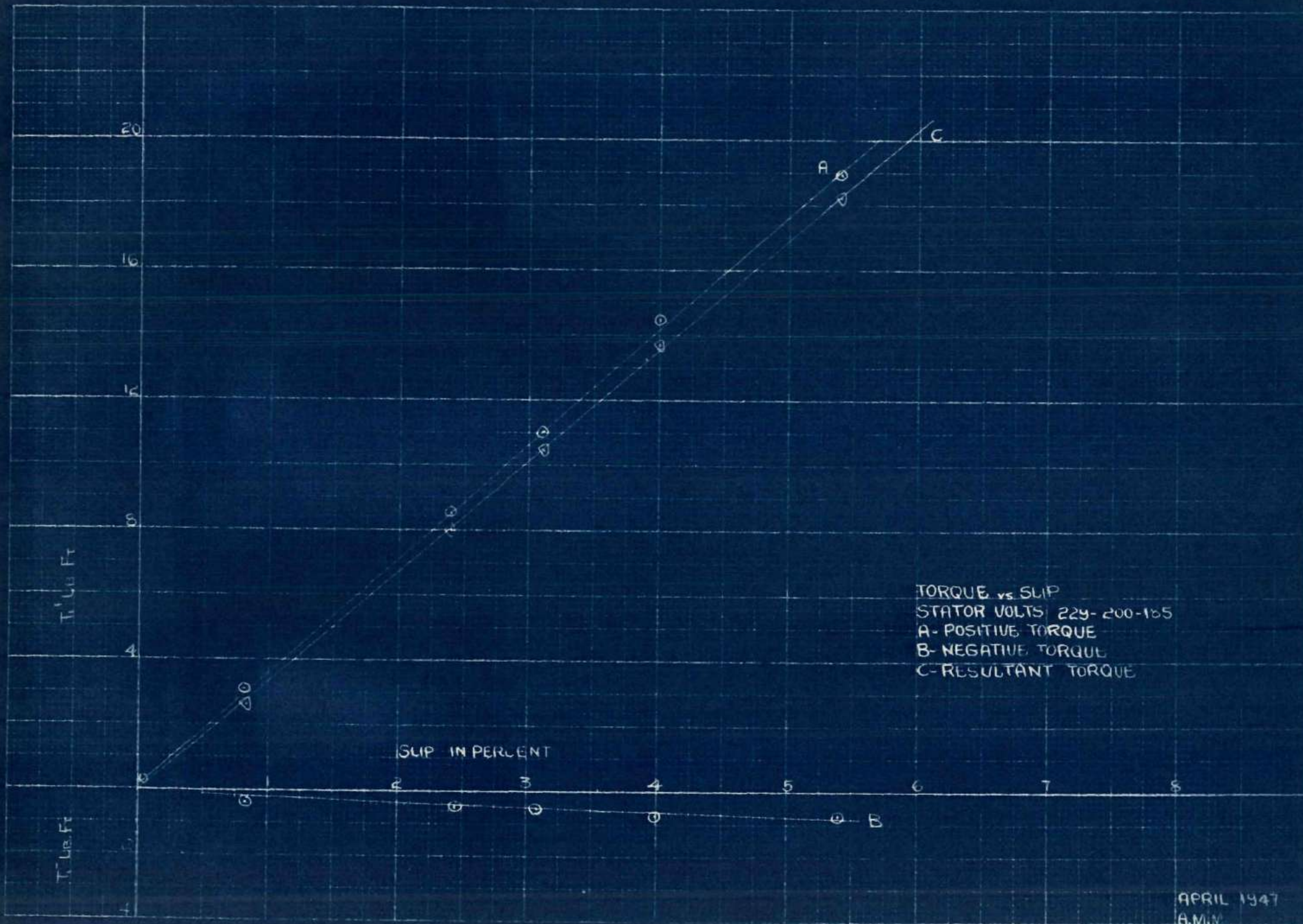

TORQUE vs. SLIP
STATOR VOLTS 205-211-236
A-POSITIVE TORQUE
B-NEGATIVE TORQUE
C-RESULTANT TORQUE
SLIP IN PERCENT
T, LB. FT.
T, LB. FT.
40
32
24
16
8
0
8
1
2
3
4
5
6
7
8
A
B
C
APRIL 1947
A.M.M.



20
16
12
8
4
4
T_1^+ IN LB FT
T_1^- IN LB FT
SLIP IN PERCENT
0
2
3
4
5
6
7
8
A
B
C
TORQUE vs SLIP
STATOR VOLTS 180-184-208
A-POSITIVE TORQUE
B-NEGATIVE TORQUE
C-RESULTANT TORQUE
APRIL 1947
A.M.M.



TORQUE vs SLIP
STATOR VOLTS 187-194-220
A-POSITIVE TORQUE
B-NEGATIVE TORQUE
C-RESULTANT TORQUE
SLIP IN PERCENT
T+ LB FT
T- LB FT
A
B
C
20
16
12
8
4
2
4
2
3
4
5
6
7
8
APRIL 1941
A.M.M



20
16
12
8
4
T_1 Lb Ft
T_2 Lb Ft
SLIP IN PERCENT
1
2
3
4
5
6
7
8
A
B
C
TORQUE vs SLIP
STATOR VOLTS 229-200-185
A- POSITIVE TORQUE
B- NEGATIVE TORQUE
C- RESULTANT TORQUE
APRIL 1947
A.M.V.

TEST DATA

No Load Test

V_{12}	V_{13}	V_{23}	I_1	I_2	I_3	W_{12}	$W_{\bar{2}3}$	W
220	220	220	8.8	8.8	9.0	640	1224	584
200	200	200	7.92	7.8	8.0	568	1008	440
180	180	180	7.0	7.0	7.0	440	816	376
160	160	160	6.2	6.2	6.2	328	648	320
140	140	140	5.4	5.4	5.4	224	512	288
118	118	118	4.6	4.6	4.6	136	392	256
100	100	100	4.0	4.0	4.0	80	288	208

From the curves plotted from the above data, the friction and windage loss was found to be 170 watts and the core loss to be 310 watts.

Blocked Rotor Test

V_{12}	V_{13}	V_{23}	I_1	I_2	I_3	W_{12}	W_{23}	W	I_{sc} cos θ calculated
52	52	52	19.6	19.6	19.6	908	60	968	10.5
43	43	43	16.0	16.0	16.0	632	40	672	9.0
33	33	33	12.0	12.0	12.0	372	20	392	6.75

Stator Resistance per phase measured at 25°C. - - - 0.3 ohms.

Rotor Resistance per phase measured at 25°C. - - - 0.26 ohms.

Ratio of transformation by test:

$$a = \frac{V}{E_2} \quad \frac{VE_2^1}{V^1E_2} = \frac{115}{88} \quad \frac{115 \times 115}{88 \times 138} = 1.362$$

Load Test with Balanced Voltages on the Stator

V_{12}	V_{13}	V_{23}	I_1	I_2	I_3	W_{12}	W_{23}	W	Torque	Speed
220	220	220	10.0	9.8	9.8	1830	-150	1680	6.15	1180
220	220	220	12.5	12.5	12.5	2570	550	3120	14.2	1155
220	220	220	14.8	14.8	14.8	3220	1090	4310	20.7	1135
220	220	220	17.5	17.5	17.5	3820	1550	5370	26.1	1120
220	220	220	20.0	20.0	20.0	4400	1970	6370	30.8	1105

Motor Characteristics from the Above Data

HP Output	Iav.	P.F.%	EFF.%	Torque	Slip %
0	8.8	0	0	0	1
1.372	9.9	44	60.5	6.15	1.66
3.10	12.4	65.5	74.0	14.2	3.8
4.45	14.8	76.5	77.0	20.7	5.4
5.54	17.5	80.5	77.0	26.1	6.6
6.50	20.0	83.5	77.0	30.8	7.9

Motor Characteristics from the Circle Diagram

HP Output	I	P.F.%	EFF.%	Torque	Slip %
3.40	12.5	67.5	78.6	16.0	
4.45	14.8	74.0	79.5	20.8	3.44
5.72	17.5	80.5	80.0	27.0	4.30
6.0	18.1	80.5	80.0	28.2	4.6
6.70	20.0	81.2	81.0	31.4	5.1

Recorded Data for Unbalanced Voltage Load Runs

V_{12}	V_{31}	V_{23}	I_1	I_2	I_3	W_1	W_2	W_T	Torque	Speed
180	184	208	9.1	21.75	17.7	1070	3190	4260	19.55	1120
180	184	208	8.0	19.8	16.6	870	2870	3740	16.9	1130
180	184	208	6.75	17.3	15.2	620	2520	3140	13.9	1140
180	184	208	5.80	15.8	14.0	410	2290	2700	11.45	1147
180	184	208	4.0	13.5	13.1	90	1850	1940	7.46	1162
180	184	208	3.75	9.75	14.0	-700	1250	550	0	1200

V_{12}	V_{31}	V_{23}	I_1	I_2	I_3	W_1	W_2	W_T	Torque	Speed
205	211	236	16.75	23.5	21.25	1100	4250	5350	24.8	1120
205	211	236	10.8	21.6	19.8	700	3900	4600	20.8	1131
205	211	236	5.25	19.6	18.1	320	3250	3570	15.9	1148
205	211	236	1.75	17.25	17.0	70	2675	2745	11.55	1160
205	211	236	1.05	16.5	16.2	24	2276	2300	9.0	1168
205	211	236	4.75	13.6	14.8	-900	1525	625	0	1200

V_{12}	V_{31}	V_{23}	I_1	I_2	I_3	W_1	W_2	W_T	Torque	Speed
187	194	220	7.15	21.65	18.5	775	3400	4175	19.1	1130
187	194	220	4.4	19.7	17.5	509	3091	3600	16.15	1138
187	194	220	2.25	17.35	16.5	250	2650	2900	12.50	1150
187	194	220	1.0	15.6	15.8	281	2129	2410	9.53	1159
187	194	220	.98	12.9	15.0	-350	1800	1450	4.44	1178
187	194	220	4.5	11.0	15.5	-820	1450	630	0	1200

V_{12}	V_{31}	V_{23}	I_1	I_2	I_3	W_1	W_2	W_T	Torque	Speed
213	209	224	15.25	21.15	22.0	3240	2780	6020	29	1104
213	209	224	14.50	20.0	20.5	3050	2520	5570	26.8	1110
213	209	224	12.8	18.2	18.6	2569	2381	4950	23.5	1116
213	209	224	10.2	15.5	15.2	2080	1470	3550	16.3	1140
213	209	224	8.0	13.25	12.6	1480	770	2250	9.17	1160
213	209	224	5.8	11.0	9.5		0	0	0	1200

V_{12}	V_{31}	V_{23}	I_1	I_2	I_3	W_1	W_2	W_T	Torque	Speed
211	204	221	16.5	22.75	17.7	2075	3750	5825	28.1	1108
211	204	221	15.1	20.6	15.6	1900	3260	5160	24.6	1114
211	204	221	11.6	17.7	13.5	1285	2755	4040	18.52	1133
211	204	221	8.1	14.3	11.7	450	2000	2450	10.35	1158
211	204	221	6.75	12.2	10.85	0	1450	1450	4.7	1177
211	204	221	6.15	10.5	10.85	0	0	0	0	1200

V_{12}	V_{31}	V_{23}	I_1	I_2	I_3	W_1	W_2	W_T	Torque	Speed
229	200	185	21.75	23.2	3.75	3840	350	4190	18.7	1135
229	200	185	20.2	19.6	3.40	3150	-100	3050	12.7	1152
229	200	185	19.2	17.5	3.5	2100	-300	2400	9.25	1163
229	200	185	18.85	16.25	3.75	2550	-570	1980	6.95	1172
229	200	185	18.70	13.25	6.1	2070	-1000	1070	1.465	1190
229	200	185	18.70	12.5	7.0	1875	-1160	715	0	1200

Calculated Data for Unbalanced Voltage Load Runs

Stator Volts 180-184-208

Iav.	Win.	Wout.	Eff.%	Slip %	HP	Torque
16.18	4260	3121	73.4	6.6	4.175	19.55
14.80	3740	2714.5	72.5	5.85	3.63	16.90
13.08	3140	2255	71.8	5.0	3.02	13.90
11.87	2700	1863	69.0	4.4	2.50	11.45
10.30	1940	1237.7	63.75	3.16	1.655	7.46
9.17	550	0	0	0	0	0

Stator Volts 205-211-236

Iav.	Win.	Wout.	Eff.%	Slip %	HP	Torque
20.5	5350	3942	73.6	6.6	5.28	24.8
17.4	4600	3360	73.2	5.7	4.50	20.8
14.31	3570	2598	72.6	4.33	3.47	15.9
12.0	2745	1900	69.1	3.33	2.55	11.55
11.25	2300	1492	65.0	2.70	2.00	9.0
11.05	625	0	0	0	0	0

Stator Volts 187-194-220

Iav.	Win.	Wout.	Eff.%	Slip %	HP	Torque
15.76	4175	3065.5	73.0	5.85	4.11	19.1
13.86	3600	2615	72.5	5.20	3.50	16.15
12.03	2900	2047	70.0	4.18	2.74	12.50
10.80	2410	1567	65.0	3.40	2.10	9.53
9.63	1450	744	51.0	1.835	0.995	4.44
10.30	630	0	0	0	0	0

Stator Volts 213-209-224

Iav.	Win.	Wout.	Eff.%	Slip %	HP	Torque
19.46	6020	4555	75.5	8.0	6.1	29.0
18.30	5570	4226	75.5	7.5	5.65	26.8
16.50	4950	3730	75.5	7.0	5.0	23.5
13.63	3550	2638.8	74.0	5.0	3.52	16.30
11.28	2250	1520	67.4	3.34	2.03	9.17
8.8	550	0	0	0	0	0

Stator Volts 211-204-221

Iav.	Win.	Wout.	Eff.%	Slip%	HP	Torque
18.98	5825	4422	75.7	7.66	5.92	28.1
17.10	5160	3918	75.4	7.05	5.24	24.6
14.26	4040	2984	74.0	5.60	4.0	18.52
11.45	2450	1700	69.5	3.50	2.28	10.35
10.10	1450	788.6	54.4	1.915	1.055	4.70
9.17	750	0	0	0	0	0

Stator Volts 229-200-185

Iav.	Win.	Wout.	Eff.%	Slip %	HP	Torque
16.2	4190	3017.5	72.0	5.41	4.04	18.7
14.4	3050	2018.5	68.1	4.0	2.79	12.7
13.4	2400	1510	63.0	3.1	2.05	9.25
12.95	1980	1157.8	58.5	2.4	1.55	6.95
12.7	1070	248.5	23.2	.834	.332	1.465
12.7	715	0	0	0	0	0

Variation of Current with Voltage Unbalance

Stator Volts	Per Cent of Max. Volts above Min.	I in High Phase at 4 h.p.	Per Cent of Max. I above Balanced Value
229-200-185	23.5	23.2	63.1
187-194-220	17.5	21.3	50.5
180-184-208	15.5	21.0	48.0
205-211-236	15.0	20.6	45.2
211-204-221	8.1	17.85	26.0
220-220-220	0	14.2	0

Variation of Efficiency with Voltage Unbalance

Stator Volts	Per Cent of Max. Volts above Min.	Eff. at 5 h.p.	Per Cent Efficiency to Balanced Efficiency
229-200-185	23.5	72	93.5
180-184-208	15.5	73	94.8
211-204-221	8.1	75	97.4
213-209-224	7.1	75.5	98.0
220-220-220	0	77.0	100

Single-Phase Operation

Test Data

V	I	Win.	Torque	Speed
220	21.2	3350	13.8	1150
220	19.0	2840	11.7	1158
220	17.0	2270	8.76	1170
220	15.2	1570	5.01	1180
220	14.5	1020	1.915	1190
220	14.0	570	0	1200

Calculated Data

HP Output	Efficiency	Torque	Slip	I
3.10	70.3	13.8	4.16	21.2
2.58	67.9	11.7	3.50	19.0
1.955	64.1	8.76	2.50	17.0
1.128	53.6	5.01	1.665	15.2
0.434	31.75	1.915	0.835	14.5
0	0	0	0	14.0

Positive- and Negative-Sequence Components of Stator Voltages

Stator Voltages	V_1^+	V_1^-
213-209-224	124.3	5.26
211-204-221	122.2	5.20
205-211-236	124.8	11.2
180-184-208	108	9.5
187-194-220	117	12.08
229-200-185	118	15.10

Characteristic Curves Calculated by the Method of Symmetrical Components

Stator Volts 213-209-224

% Slip	HP	Torque	Eff.	I_1^+	T_1^+	T_1^-	I_1^+ by test
8.0	5.94	28.25	75.5	18.37	29.4	0.1	18.75
7.5	5.72	27.0	75.7	17.65	28.2	0.1	17.95
7.0	5.45	25.65	75.9	16.90	26.8	0.1	16.30
5.4	4.05	19.20	75.0	13.80	22.0	0.1	13.42
3.34	2.70	12.20	72.2	11.18	13.64	0.1	11.10
0	0	0	0	8.8	0	0	8.8

Stator Volts 211-204-221

% Slip	HP	Torque	Eff.	I_1^+	T_1^+	T_1^-	I_1^+ by test
7.66	5.6	26.6	75.5	17.62	27.8	0.09	18.42
7.05	3.24	24.6	75.5	16.60	25.75	0.09	16.55
5.6	4.14	20.0	75.5	13.90	21.0	0.09	14.0
3.5	2.78	12.63	72.5	11.13	13.8	0.088	10.94
1.915	2.10	9.82	68.0	10.76	10.45	0.083	9.36
0	0	0	0	8.8	0	0	8.8

Stator Volts 205-211-236

% Slip	HP	Torque	Eff.	I_1^+	T_1^+	T_1^-	I_1^+ by test
6.6	5.1	23.95	73.5	16.2	25.3	0.41	18.4
5.7	4.45	20.70	73.0	13.7	22.4	0.41	15.6
4.33	3.46	15.85	70.9	12.6	17.3	0.408	13.12
3.33	2.72	12.30	67.8	11.2	13.76	0.407	11.0
2.70	2.15	10.0	64.0	10.3	11.2	0.406	10.2
0	0	0	0	8.8	0	0	8.8

Stator Volts 180-184-208

% Slip	HP	Torque	Eff.	I_1^+	T_1^+	T_1^-	I_1^+ by test
6.6	3.82	17.9	72.5	14.03	19.2	0.296	15.1
5.85	3.36	15.6	70.4	12.92	16.9	0.296	13.9
5.0	2.84	13.08	68.9	11.68	14.42	0.296	12.5
4.4	2.45	11.2	67.0	10.95	12.50	0.295	10.92
3.16	1.87	8.45	63.0	9.47	9.75	0.295	9.8
0	0	0	0	8.8	0	0	8.8

Stator Volts 187-194-220

% Slip	HP	Torque	Eff.	I_1^+	T_1^+	T_1^-	I_1^+ by test
5.85	3.95	18.32	70.7	14.0	19.88	0.478	14.67
5.20	3.59	16.54	70.2	13.12	18.10	0.477	12.34
4.18	2.96	13.50	68.0	11.70	15.05	0.476	10.0
3.40	2.31	10.20	64.0	10.0	12.40	0.476	9.0
1.835	1.128	5.03	49.5	8.55	6.48	0.466	8.0
0	0	0	0	8.80	0	0	8.8

Stator Volts 229-200-185

% Slip	HP	Torque	Eff.	I_1^+	T_1^+	T_1^-	I_1^+ by test
5.41	3.73	17.25	67.8	13.52	18.95	0.746	14.81
4.0	2.78	12.70	64.0	11.50	14.46	0.744	13.10
3.1	2.15	10.0	60.0	10.60	11.0	0.50	11.88
2.4	1.70	8.0	54.0	9.80	8.5	0.50	11.26
0.834	0.398	1.759	16.0	8.74	3.174	0.411	9.30
0	0	0	0	8.80	0	0	8.8

XIV DISCUSSION OF THE CURVES

The circle diagram was drawn to obtain a rough check on the motor performance. It was drawn from the data obtained from the no load and blocked rotor runs. The line current, torque, efficiency, slip, and power factor for different loads were read off the circle diagrams and plotted against the horsepower output. In each case the equivalent curve obtained from actual test was plotted on the same sheet for comparison. The torque curve obtained from the circle diagram coincides with that from test. The curves for efficiency, power factor, and line current check to within two and a half per cent with the equivalent curves as obtained from actual test. Only the slip curves show some real discrepancy. This is undoubtedly due to the scale chosen and to the fact that it is hard to read off slip from the circle diagram because of the small measurements involved. It must be remembered, however, that the circle diagram is only an approximate method and the results obtained are therefore quite satisfactory. They gave a check on the motor performance and served as a starting point for the other tests.

Curves for Unbalanced Voltages

Innumerable load tests with different degrees of voltage unbalance were made on the motor. However, the greatest unbalance that could be obtained was a 24 per cent unbalance, the voltages on the stator being 229, 200, and 185. Beyond that point the current in the high phase of the motor reached upward of 125% of full load current at practically no load, and, therefore, a load run could not be made. From all the tests that have been made, six representative runs were chosen and the respective curves drawn. The curves were chosen so as to give the most illuminating comparison with those drawn for balanced conditions. In each case the curves for balanced and for unbalanced voltage conditions were drawn on the same sheets.

Currents

In the first place the average currents for the various unbalanced voltage runs were calculated and plotted against horsepower output. The line current curve for balanced voltage conditions was plotted on the same sheet. Two sheets were used, each having three curves for various degrees of voltage unbalance, in order to facilitate comparison. It is seen from these curves that for each and every degree of voltage unbalance, the average line current is higher than the corresponding value under balanced voltage conditions. This is important because it shows that for any degree of voltage unbalance the copper loss in the stator is higher than that for balanced conditions, for the same horsepower output.

Next, the line currents in the various phases of the motor were plotted against horsepower output for all six runs. These six sets of curves show that for any degree of voltage unbalance the line currents in two of the motor phases are higher than the corresponding currents for balanced conditions, while the current in the third phase is lower. The curves also show a tendency for the amount of unbalance in line currents to become greater as the load decreases. This is because the current for the positive-sequence system decreases as the motor speed increases with decrease in load, while the current for the negative-sequence system increases. The amount of current taken by the motor with the negative-sequence voltage applied with the rotor running against the normal direction of rotation is inversely proportional to the internal impedance of the motor. Hence, motors with high impedance, which is evidenced by low pull-out torque, will have less unbalance in currents than those with low impedance.

Finally a curve was plotted of the percentage increase of the current in the high phase with respect of its equivalent value under balanced conditions, against the degree of voltage unbalance. By the degree of voltage unbalance is meant the percentage of the maximum above the minimum volts applied to the stator for each run. From this curve it can be seen that for a 24 per cent unbalance in the voltages the current in the high phase is 63 per cent higher than itscorresponding value for balanced conditions. For a 15 per cent unbalance in voltages, which is the more likely value to be encountered in practice, the current in the high phase is about 50 per cent higher than its corresponding value for balanced conditions.

Torque

As was explained in the theory part of this paper the negative torque produced by the negatively rotating voltage subtracts from the torque produced by the positive voltage, thus resulting in a lower torque for the motor as compared to the torque under balanced conditions. The torque curves plotted do show this decrease in torque, but they also show that this decrease is rather insignificant. This fact can be explained thus. As the torque varies as the square of the voltage, if the torque T is known for the positive-sequence voltage, for any slip S, the resultant torque for the unbalanced voltage is

$$T - T \left[\frac{\text{negative-sequence voltage}}{\text{positive-sequence voltage}}\right]^2$$

In practice, when the motor is operating on unbalanced voltages and the highest applied voltage does not exceed the lowest applied voltage by more than 15 per cent, the negatively rotating voltage is always less than 10 to 15 per cent of the positively rotating voltage. Since the torque varies as the square of the voltage, the negative torque will not exceed 1 to 2 per cent of the positive torque and can be neglected. The torque curves clearly show this fact.

Efficiency and Slip

The efficiency curves for all the six runs show a decrease in efficiency with unbalanced voltages on the stator. The negative torque produced by the negatively rotating field subtracts from the torque produced by the positively rotating field. This represents so much lost torque and output, resulting in lower efficiency. It was shown in the discussion of the current curves that the average of the unbalanced currents is always greater than the corresponding current for balanced conditions. This gives a higher copper loss in the stator, proportional to $(I^+)^2$ and $(I^-)^2$.

As was explained above, the torque decreases slightly when the voltages applied to the stator are unbalanced. To produce a given amount of torque, the rotor must slip more. This is clearly shown in the slip curves where all the six curves lie above the slip curve for balanced voltage conditions. Since the rotor copper loss is equal to the rotor input multiplied by the slip, this increase in slip with unbalanced voltages results in a higher copper loss in the rotor. The unbalanced voltages in the stator also tend to distort the field, causing a slight increase in the iron losses. All these causes tend to increase the input for a given output, and thus reduce the efficiency.

A curve was plotted of the efficiency under unbalanced conditions expressed as a percentage of the corresponding efficiency under balanced conditions against the percentage of voltage unbalance. The curve comes out to be a straight line, showing a linear decrease in efficiency with an increase in the degree of voltage unbalance.

Heating

Although no specific tests were made on the heating of the motor, certain deductions can be made from the curves already plotted. It has been shown that for a 15 per cent unbalance in voltages the current in the high phase is 50 per cent higher than the corresponding current for balanced conditions. Since the heat produced in a coil varies as the square of the current, it is seen that the coils of the high phase heat up 2.25 times as much as the coils under balanced conditions for the same output. The amount of heat to be dissipated by the motor is also increased due to the increase in losses and the decrease in efficiency.

However, there is another fact to be considered. In the curves of the various phase currents it is seen that the currents in two of the phases are higher, while the current in the third phase is considerably lower than the corresponding currents under balanced conditions. As the coils in the motor are distributed around the stator in phase groups, the coils of the high phase will have on either side of them groups of coils carrying lower currents. Thus, there is a tendency for the heat to flow along the stator core from the hot coils to the cooler coils on either side, thus equalizing the heating. There is, however, a definite temperature gradient from the coil in the center of the hot group to the cooler groups on either side.

The motor is given a horsepower rating such that for continuous operation at that output the temperature rise does not exceed 50°C. From the above discussion it is obvious that the motor will heat up more under unbalanced voltage conditions than for balanced voltages for the same horsepower output. For the same temperature rise, therefore, the rating of the motor with unbalanced stator voltages should be less than its rating for balanced voltages. For a voltage unbalance of 15 per cent the current in the high phase is 50 per cent above the corresponding current for balanced conditions. Taking into account the fact that heat will flow from the hot coils to the cooler coils on each side, it can be assumed that for this degree of unbalance the motor rating should be decreased about 25 per cent, or else a motor with a sufficient margin in temperature to take care of a 25 per cent overload should be used.

Single-Phase Test Curves

The characteristic curves for the single-phase load run are quite similar to those for the unbalanced voltages. This,of course, is to be expected since operating a three-phase motor on single phase is just a special case of operating it with unbalanced stator voltages.

The motor was started up with three-phase, balanced voltages on the stator. It was loaded up to rated output, i.e., 6.5 hp. with 20 amperes rated current per phase. Then one of the power lines was opened. The output of the motor suddenly dropped to 3.1 hp., while drawing 21.2 amperes from the line. The slip increased while the efficiency decreased correspondingly. The torque remained practically the same.

For an output of 3.1 hp., the following results were read from the plotted curves for the single-phase run.

Item	Three-Phase Operation	Single-Phase Operation	Single-Phase in % of Three-Phase
Line Current	12.5	21.2	170.0
Slip	3.75	4.16	111.0
Efficiency	73.0	70.3	96.3
Torque	13.9	13.8	100.0

For ordinary values of slip the effect of the negative-sequence power and torque is very small. This was explained above for the case of the unbalanced stator voltages, and is true for this case also. The reduction in the power output suffered by the motor during single-phase operation, assuming the same slip as for three-phase operation, is primarily due to the decrease of the positive-sequence power. The positive-sequence current, when the motor is operated single-phase, is less than the current taken during three-phase operation on balanced voltages of the same magnitude as the single-phase voltage. This is true since, obviously,

$$\frac{V}{\sqrt{3}\,(Z^{+} + Z^{-})} < \frac{V}{\sqrt{3}\,Z^{+}}$$

Since the power is proportional to the square of the current it follows, therefore, that the positive-sequence power is lower for the single-phase operation than for the three-phase.

If a three-phase motor is operated on rated line voltage with one line terminal open, the core loss in the stator is somewhat less than when the motor operates with balanced rated line voltages, because the maximum flux density in the stator decreases as the field revolves from a maximum along the axis of the stator winding to a minimum at right angles to this axis. With balanced impressed voltages, the maximum flux density in the stator does not vary as the field revolves. With balanced impressed voltages, the rotor core loss is small under load conditions because of the small slip of the rotor with respect to the revolving field. This is also true under the conditions of single-phase operation in regard to the core loss produced in the rotor by the positive-sequence flux. Although the flux produced by the negative-sequence magnetizing currents is small under operating conditions, the rotor core loss produced by this flux is not negligible because the rotor slip with respect to the negative-sequence flux is (2-S), which is nearly 2 for operating values of slip. Although the stator core loss of a three-phase induction motor, operating with fixed impressed voltage and frequency, with one line terminal open, is slightly less than when it is operating polyphase; this decrease in stator core loss is balanced in part by the increase in the rotor core loss due to the double-frequency negative-sequence flux in the rotor. For this reason, the total core loss of the polyphase motor when operating with one line terminal open is not greatly different than when operating with balanced impressed voltages.

The decrease in efficiency for the single-phase operation is due primarily to the increased copper losses, both in the stator and in the rotor.

Calculated Curves

For each of the load runs made with unbalanced voltages on the stator, the characteristic curves were calculated from the motor constants by the method of symmetrical components. A sample calculation of one curve will be given in the appendix. In each case the calculated curve is plotted on the same sheet with the corresponding curve obtained from actual test, in order to compare the two together.

Current

For each load run, the line currents as obtained from test were resolved into their symmetrical phase components and the positve-sequence components plotted against horsepower output. These are the test curves. The positive-sequence components of the line currents were then calculated from the motor constants, using the same value of slip as obtained from test, and plotted against horsepower output on the same sheets as the test curves. As can be seen from the curves, the results are very satisfactory. In five of the load runs the calculated currents check to within 2 per cent of the test value; in the sixth run they check to within six per cent.

Efficiency

Five of the calculated efficiency curves check to within 4 per cent of the test curves. The sixth calculated curve checks to within 6.5 per cent.

Torque

All the six calculated torque curves check to within 2 per cent of the test curves.

Positive and Negative Torque

For every load run the torque due to the positive-sequence voltage, and the torque due to the negative-sequence voltage were calculated for various values of slip and plotted against horsepower output. In no case does the negative-sequence torque exceed the positive-sequence torque by more than 2 per cent. This shows that the unbalance in voltages that may occur in service will generally have a negligible effect upon the torque characteristics.

Slip

All the calculated slip curves check to within 6 per cent or closer of the test curves.

XV SUMMARY OF RESULTS

When polyphase induction motors are required to operate from circuits in which the voltages on the different phases are unequal, the degree of voltage unbalance usually encountered is from 10 to 20 per cent. For such unbalance:

a. The effect on torque is practically negligible.

b. The efficiency is reduced from 1 to 6 per cent of the efficiency under normal balanced conditions.

c. The current in the high phase is increased from 30 to 55 per cent above the current under normal balanced conditions.

d. The slip is increased from 5 to 20 per cent above the slip under normal balanced conditions.

e. To take care of the heating in the coils carrying the highest current, the horsepower output of the motor should be reduced from 15 to 30 per cent, or motors with a sufficient margin in temperature to take care of from 15 to 30 per cent overload should be used.

If the motor constants can be determined accurately, the method of symmetrical components can be used to calculate the motor characteristics, under any conditions of voltage unbalance, with considerable accuracy. In engineering practice a check to within 8 per cent between test and calculated values is considered very good. In this case the two results checked to within 5 per cent.

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Appendix

Calculations of the Motor Performance by the Method of Symmetrical Components

Stator volts 213-209-224 volts

$r_{a1} = 0.506$ ohms

$r_{a2} = 0.506$ ohms

$r^{+}_{b1} = 0.575$ ohms

$r^{-}_{b2} = 1.035$ ohms

$X_{a1} = X_{a2} = 0.64$ ohms

$X_{b1} = X_{b2} = 0.64$ ohms

Core loss = 310 watts

F + W loss = 170 watts

$I_n = 8.8$ amperes

$g_n = \frac{310}{3X(127)^2} = 0.00813$

P.f. at no load $= \frac{584}{\sqrt{3}X220X8.8} = 0.174$

$\theta = 79.97^\circ$

$\sin\theta = 0.984$

$I_\phi = 8.8 \times .984 = 8.65$

$b_n = \frac{8.65}{127} = 0.0680$

$Y_\phi = 0.00813 - j\,0.0680 = 0.0685 \underline{\backslash 83^\circ 11'}$

$$Z_\phi = \frac{1}{0.0685 \underline{\backslash 83^\circ 11'}} = 14.6 \overline{/ 83^\circ 11'} = 1.73 + j\,14.5$$

The next step is to resolve the stator voltages into their symmetrical phase components.

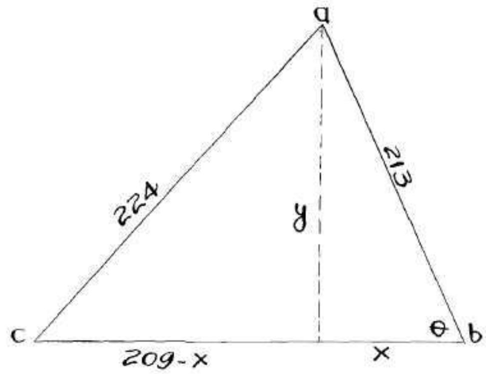

Figure 1

$$Y^2 = 213^2 - X^2 = 45{,}400 - X^2$$

$$Y^2 = 224^2 - (209 - X)^2 = 50100 - (43{,}600 - 418\ X + X^2)$$

$$= 6500 + 418\ X - X^2$$

$$45{,}400 - X^2 = 6500 + 418\ X - X^2$$

$$418\ X = 38900$$

$$X = 93$$

$$\cos\theta = \frac{93}{213} = 0.436$$

$$\theta = 64.15^\circ$$

The positive-sequence potential to neutral V_1^+ is:

$$V_1^+ = \frac{1}{3}(V_{ab} + V_{bc}\ \underline{|60^\circ})$$

$$V_1^+ = \frac{1}{3}\left[213 + 209 \times .561 + j\ 209 \times .827\right] = \frac{373}{3} = 124.3 \text{ volts}$$

The negative-sequence potential to neutral V_1^- is:

$$V_1^- = \frac{1}{3}(V_{ab} + V_{bc}\ \overline{|60^\circ})$$

$$V_1^- = \frac{1}{3}\left[213 - 209 \times .997 + j\ 209 \times .07237\right] = \frac{15.77}{3} = 5.26 \text{ volts}$$

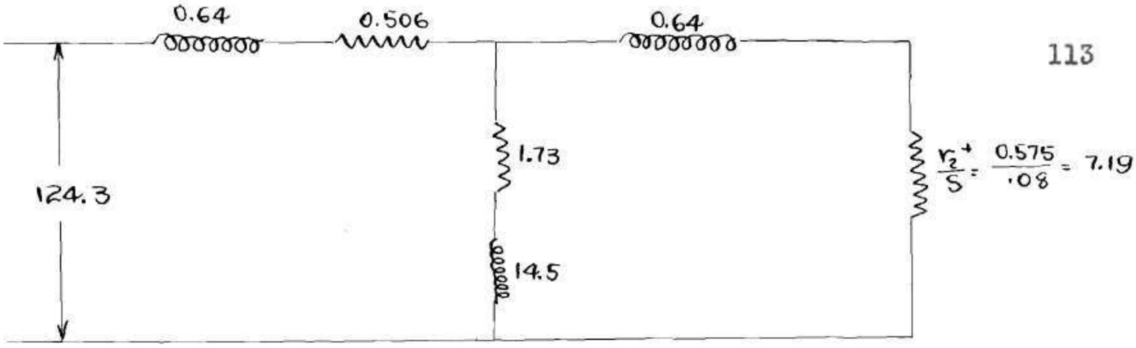

Figure 2

$$Z_1 = 0.506 + j\,0.64 + \frac{(1.73 + j\,14.5)(7.19 + j\,0.64)}{(8.92 + j\,15.14)} .$$

$$Z_1 = 6.77 \text{ ohms.}$$

The negative-sequence vector diagram is:

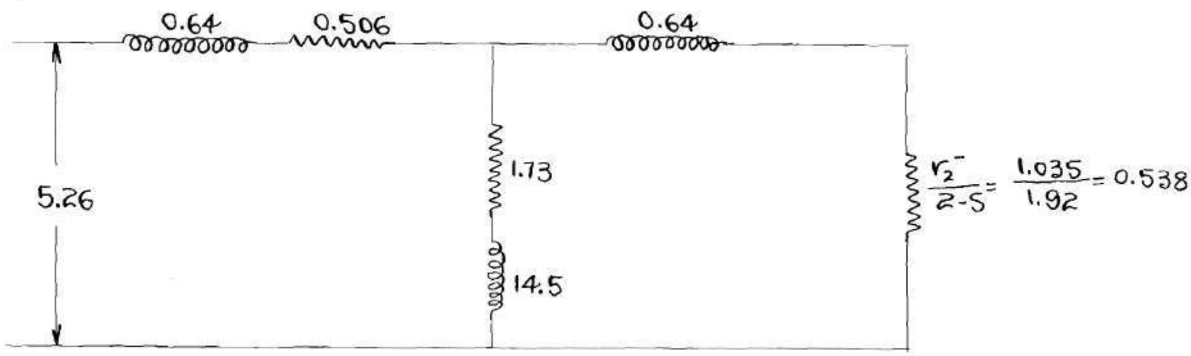

Figure 3

$$Z_2 = 0.506 + j\,0.64 + \frac{(1.73 + j\,14.5)(0.538 + j\,0.64)}{2.268 + j\,15.14}$$

$$= 1.612 \text{ ohms.}$$

$I_1^+ = \dfrac{V_1^+}{Z_1} = \dfrac{124.3}{6.77} = 18.37$ amperes compared to 18.75 amperes as obtained by test.

$I_1^- = \dfrac{V_1^-}{Z_2} = \dfrac{5.26}{1.612} = 3.26$ amperes

$$I_2^{+} = \frac{18.37 \times 14.6}{17.55} = 15.27 \text{ amperes}$$

$$I_2^{-} = \frac{3.26 \times 14.6}{15.3} = 3.12 \text{ amperes}$$

$$P_2^{+} = (I_2^{+})^2 r_2^{+} \frac{1 - S}{S} = (15.27)^2 \times 0.575 \times \frac{0.92}{0.08} = 1540 \text{ watts}$$

$$P_2^{-} = (I_2^{-})^2 r_2^{1} \frac{1 - S}{2-S} = (3.12)^2 \times 1.035 \times \frac{0.92}{1.92} = 4.83 \text{ watts}$$

$$P = P_2^{+} - P_2^{-} = 1540 - 4.83 = 1535.17 \text{ watts}$$

Total pulley power $P_p = 3 \times 1535.17 - 170 = 4435.5$ watts.

HP output $\frac{4435.5}{746} = 5.94$ compared to 6.1 hp obtained from test.

Torque $\frac{5.94 \times 5250}{1104} = 28.25$ pounds feet compared to 29 pounds feet obtained from test.

Cu loss per phase:

$(18.37)^2 \times .506 = 170.6$

$(3.26)^2 \times .506 = 5.38$

$(15.27)^2 \times .575 = 134.0$

$(3.12)^2 \times 1.035 = 10.0$

Total $= 319.98$

Total copper loss $3 \times 319.9 = 960$ watts

Total loss $= 960 + 310 + 170 = 1440$ watts

Efficiency $= \frac{4435.5}{4435.5 + 1440} = 75.5\%$ as compared to 75.5% obtained from test.

$$T^{+} = \frac{3 \times 1540 \times 5250}{746 \times 1104} = 29.4 \text{ pounds feet}$$

$$T^{-} = \frac{3 \times 4.83 \times 5250}{746 \times 1104} = 0.1 \text{ pounds feet}$$

Calculated Constants from Previous Data

Equivalent Z per phase	1.53 ohms
Equivalent R per phase	0.838 ohms
Equivalent X per phase	1.280 ohms

Effective resistance of stator per phase @ 25°C. and 60 cycles:

$$.838 \times \frac{.300}{.300 + .260} = 0.448$$

Ohmic resistance of stator per phase @ 75°C.:

$$.300(1 + 50 \times .00385) = .3578 \text{ ohms}$$

Effective resistance of stator per phase @ 75°C.:

$$.3578 + .448 - .300 = .506 \text{ ohms}$$

Ohmic resistance of rotor per phase referred to stator @ 25°C.:

$$.260 \times (1.362)^2 = 0.482 \text{ ohms}$$

Ohmic resistance of rotor per phase referred to stator @ 75^{2}C.:

$$.482(1 + 50 \times .00385) = .575 \text{ ohms}$$

Effective resistance of rotor per phase referred to stator @ 75°C. and 120 cycles:

$$.575 \times 1.8 = 1.035 \text{ ohms}$$

The above method of calculating resistances is due to R. R. Lawrence as shown in his book on "Alternating Current Machinery".